Optimal Dynamic Inspection

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Abstract

We study a discounted repeated inspection game with two agents and one principal. Both agents may profit by violating certain rules, while the principal can inspect on at most one agent in each period, inflicting a punishment on an agent who is caught violating the rules. The goal of the principal is to minimize the discounted number of violations, and he has a Stackelberg leader advantage. We characterize the principal's optimal inspection strategy.

1 Introduction

Inspection games model situations in which a *principal* verifies that some *agents* adhere to certain legal rules. Each agent may benefit from violating the rules, yet by doing so he faces a penalty if the violation behavior is observed by the principal. Such situations are prevalent in modern life; for example, the tax authority audits tax payers, and the environmental protection agency inspects on firms who produce air or water pollution. Typically the resources of the inspection agencies are limited, hence they cannot inspect on all agents at all times. Consequently, the question regarding the identification of the optimal inspection scheme arises.

One way to deter agents from violating the rules is by imposing an enormous fine for every detected violation. Nevertheless, as noted by Harrington (1988), in reality the fines of discovered violations is typically low. The reasons involve legal constraints (in most states in US there is a restriction on the size of penalties that can be levied on a firm each day) and ethical considerations (Becker (1968)). Moreover, if the adjudication is not perfect, a large fine may cause very high loss on social welfare since innocent inspectees can be convicted

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(Franzoni (2000)). Therefore, it is reasonable to treat the amount of fine as an exogenous variable and address the following question: given the fixed penalty level and the limited inspection resources, what is the optimal inspection scheme?

A large literature on optimal auditing strategies is static (e.g., Reinganum and Wilde (1985), Cremer, Marchand, and Pestieau (1990), Sanchez and Sobel (1993)). These authors restricted the probability of audit to be independent of a taxpayer's previous compliance history (although this probability can be contingent on the taxpayer's reported income).

Landsberger and Meilijson (1982) first proposed a *dynamic state dependent inspection scheme* in auditing, which can attain a more efficient use of resources compared with the static schemes. This scheme audits individuals, who discounts future payoffs, with different probabilities depending on their compliance history. An agent who is inspected and found adhering is assigned to state 1(where he faces low inspection probability) and an agent who is found violating is assigned to state 2 (where he faces high inspection probability). Under this scheme, in equilibrium, agents in state 1 violate the rules and agents in state 2 adhere. Consequently any agent that is inspected is moved to the other state. We can view state 1 as a *rewarding state*, and state 2 as a *neutral state*. The inspector chooses the auditing probabilities to minimizes the expected number of agents in the rewarding state while keeping incentive compatibility.

Greenberg (1984) argued that the strategy proposed by Landsberger and Meilijson is not optimal, and Greeberg suggested an improved inspection scheme. He added to the inspection scheme a third state which serves as a *punishment state*, from which no escape is allowed, so that once in this state, one faces a sure audit in the future. Greenberg showed that, if players are extremely patient, then regardless of how small the percentage of individuals the tax authorities can audit (which is determined by the agency's budget), the rate of violations can be made arbitrarily small. This is because when players *do not* discount future payoffs, the loss from the risk of being moved to the punishment state, no matter how small the probability is, always exceeds the benefit from the current-period gain from an undetected violation.

Even though the two (or three) states dependent inspection schemes have drawn lots of attention in the literature (for instance, Harrington (1988), Harford and Harrington (1991), and Harford (1991) adapted these auditing mechanisms into environmental control problems to explain the phenomenon of high compliance in the absence of strict enforcement), as acknowledged by Greenberg, it is far from being optimal if players are *not* infinitely patient. An obvious way for improvement is adding more states to the model. For instance, an agent will be moved to a "rewarding" state only if he is found compliant for a certain number of times. Moreover, there may exist inspection schemes which are non-state-dependent and outperform all state-dependent schemes. This paper is part of a general agenda to identify the optimal inspection schemes.

We study a model with one inspector and two agents. There are infinite number of periods, at each period each agent has two possible actions, V (violate) or A (adhere) and the inspector can inspect on at most one agent. It is assumed that the inspector will find out the action of an agent if and only if this agent is inspected upon. At each period, an agent who plays A obtains 0. The agent who plays V gets 1 if not being inspected and $-c$ otherwise. If $c > 1$, the inspector can deter both agents from violating in the stage game by inspecting on each one of them with probability 0.5 and the optimal inspection scheme is trivial. Therefore we focus on the case $0 < c < 1$. For simplicity it is assumed that in case of indifference, an agent chooses A. This is as if agents violate the rules only if such a violation is profitable. As for the inspector, he loses 1 for each instance in which an agent played V, regardless of whether this agent is inspected or not. This payoff is not observable by the inspector until the end of the game. Each agent and the inspector maximizes the discounted payoff with a common discount factor δ. We assume that the inspector has *Stackelberg leadership advantage*, so that he announces and commits to an inspection scheme at the beginning of the game. Motivation for Stackelberg - rules of agency, real-life example, related literature. Note that the goal of the inspector is to minimize the discounted number of violations, with no separate value for the extraction of penalties after its detection. In many cases the damage caused by the violation behavior is incomparable with the monetary penalties because they are in different dimensions. For instance, in the environmental control problem, the damage caused by a firm's illegal emission of polluted air or water may be difficult to repair.

It can be verified that a *myopic* agent chooses A if and only if the probability of being inspected in the current period is no less than $\frac{1}{1+c}$. Since $c < 1$, we have $\frac{1}{1+c} > \frac{1}{2}$, and therefore the inspector who faces two myopic agents can deter *at most* one from violating in each period. As noted by Landsberger and Meilijson (1982) and Greenberg (1984), when the agents are *far-sighted*, the inspector can do better since future frequency of auditing can serve as a mean to discipline agents. In this paper we identify the optimal inspection scheme.

First note that there is no cheat-proof mechanism. Indeed, an agent who is inspected with probability less than $\frac{1}{1+c}$ in the first period can obtain a positive payoff by choosing Violate in the first period and Adhere in all subsequent periods. Consequently, the inspector has to tolerate at least one agent to violate in some periods. We refer to the stages in which the inspector intentionally allows an agent to violate as *rewarding stages*. Our goal is to make the best use of the rewarding stages to deter most violations.

The optimal inspection scheme consists of two phases. In Phase 1 there is a "war of tokens". At the beginning both agents have the same amount of tokens and the inspector inspects on them with the same probability (0.5). Depending on the realization of the inspector's action, the agent that is inspected upon and found adhering gains tokens, while the agent that is *not* inspected loses tokens. In the next period, the inspector inspects on the agent with more tokens with a lower probability. Again, depending on the realization of the inspector's second period action, the agent that is inspected upon gains tokens, and the other one loses tokens. The number of tokens that is added to and subtracted from the agents depend on the number of tokens they currently have, and the sum of both agents' tokens is not necessarily constant. The process continues until one of the agent loses all his tokens and Phase 1 ends. In Phase 1, any agent who is found violating faces sure audit in all future periods.

In Phase 2, the agent that has zero token is inspected with probability $\frac{1}{1+c}$ in every period and consequently chooses Adhere throughout. The other agent is inspected with the remaining probability ($\frac{c}{1+c}$), until he is actually being inspected upon. At that moment he triggers a *rewarding cycle*, which is periodic with length $k+1$ that lasts forever. At the first k stages of each cycle the agent is inspected with probability $\frac{c}{1+c}$. If he was *never* found violating in the past, then in the $(k+1)$th stage he will *not* be inspected, and will be able to obtain 1 by violating. If he is found violating in any of the first k stages, he faces sure audit in all future periods.

Several insights can be drawn. First, under the optimal inspection scheme, no agent chooses Violate in Phase 1. This is because comparing with issuing a reward (i.e., inspecting on an agent with zero probability) in Phase 1, the inspector benefits from delaying the reward and using it as a deterrence for violations. Second, the strategy in Phase 1 exhibits the feature of "upgrading the inspected agent and downgrading the uninspected one". This feature seems to be unfair to the uninspected agent, but it is inevitable: an agent's choice of Violate/Adhere depends only on the difference between his expected payoffs if he is inspected and found violating and if he is found adhering - the expected payoff of *not being inspected*, no matter how low it is, does *not* affect an agent's current period action. Consequently, the inspector, whose goal is to deter violation, would not "waste" the reward on the occurrence of uninspection. Another feature of Phase 1 is that the inspector inspects on the agent with more tokens with a lower probability. An agent with more tokens faces a higher expected payoff if he is found adhering. Therefore, even if he is inspected with a relatively low probability, he is not willing to take the risk of violating and losing his high "endowment". Finally, in the *rewarding stages* in Phase 2, that is, the $(k+1)$th stage in each cycle, the inspector does *not* inspect on the rewarded agent. The rewarding stage is designed for the agent to violate, so that he has incentive to adhere in the previous k stages of the cycle as well as to adhere in Phase 1. A positive (even if small) probability of being inspected in the rewarding stage hurts the agent's profit, which in turn reduces the number of adhering periods the rewarding stages can sustain.

By dynamically changing the probability of inspection, while having infinitely many states, our inspection scheme significantly outperforms the mechanisms proposed by Landsberger and Meilijson (1982) and Greenberg (1984). In the case where $c = 0.7$ and $\delta = 0.9$ (players are quite patient), the optimal expected payoff of the inspector within the framework of the three-state mechanism (as proposed by Greenberg (1984)) is -7.456, while in our mechanism, the inspector can guarantee an expected payoff of -0.36.

Many questions remain open - for instance, the structure of the optimal inspection mechanism when players have different discount factors; when there are more than one inspectors and more than two agents; when the adjudication is not perfect; or when agents assign different values to violation.

A related strand of literature concerns *recursive inspection games* (e.g., Dresher (1962), Maschler (1966), Avenhaus and Von Stengel (1992), Avenhaus, Von Stengel, and Zamir (2002)). These models include only one agent, whom can be inspected only m times out of n periods. Our model has two agents and there is no sure deadline - players discount future payoffs, which is equivalent to having the game end at a random time. Moreover, in our model there is no constraint on the number of periods the inspector can audit (instead, there is a resource constraint which allows the inspector to audit at most one individual in each period). Recently, repeated inspection games have drawn lots of attention in Computer Science, under the context of crime control: an attacker can choose among several sites to attack, while the defender can select only a limited number of sites to protect. Most papers in this strand of literature either assume *myopic* attackers (e.g., Letchford, Conitzer, and Munagala (2009), Blum, Haghtalab, and Procaccia (2015), Marecki, Tesauro, and Segal (2012)) or *bounded rational* attackers (e.g., Yang et al. (2014), Nguyen et al. (2013), Haskell et al. (2014), Kar et al. (2015)). In contrast, in our model the players are fully rational and the strategy takes into account all past histories.

2 Model

The stage game is given in strategic form $\Gamma = (N, (A_i)_{i\in N}, (u_i)_{i\in N})$, where $N = \{0, 1, 2\}$ is the set of players, A_i is the set of actions available to player i, and $u_i : A \to \mathbb{R}$ is the payoff function of player i in the stage game ($A \equiv \prod_i A_i$ is the set of action vectors). The inspector is referred to as Player 0, with $A_0 = \{I_1, I_2, \emptyset\}$. Here $\emptyset$ represents no inspection, I_1 and I_2 represents inspecting on Agent 1 and 2, respectively. Agents are referred to as Player 1 and

2. For $i = 1, 2$, $A_i = \{V, A\}$ and

$$u_i(a) = \begin{cases} 0 & \text{if } a_i = A \\ -c & \text{if } a_i = V \text{ and } a_0 = I_i \\ 1 & \text{if } a_i = V \text{ and } a_0 \neq I_i, \end{cases} \tag{1}$$

where $c < 1$. For the inspector,

$$u_0(a) = -\big(\mathbb{1}_V(a_1) + \mathbb{1}_V(a_2)\big), \tag{2}$$

where for $i = 1, 2$,

$$\mathbb{1}_V(a_i) := \begin{cases} 1 & \text{if } a_i = V \\ 0 & \text{if } a_i = A. \end{cases} \tag{3}$$

At the end of every stage the two agents observe the inspector's action. If the inspector monitored one of the agents at that stage, the action of the monitored agent is publicly observed. This is equivalent to the situation in which the players observe at the end of each stage a public signal y, drawn from a signal space $Y = (V_1, A_1, V_2, A_2, \emptyset)$. The realization of signal y, given the action profile $a \in A$ is deterministic and it is denoted by $y(a)$, where

$$y(a) := \begin{cases} \emptyset & \text{if } a_0 = \emptyset \\ V_1 & \text{if } a_0 = I_1 \text{ and } a_1 = V \\ A_1 & \text{if } a_0 = I_1 \text{ and } a_1 = A \\ V_2 & \text{if } a_0 = I_2 \text{ and } a_2 = V \\ A_2 & \text{if } a_0 = I_2 \text{ and } a_2 = A. \end{cases} \tag{4}$$

In the repeated game, the only public information available in period t is the $(t-1)$-period history of public signals, $h^t \equiv (y^0, y^1, ..., y^{t-1})$. The set of *public histories* is

$$\mathscr{H} \equiv \cup_{t=0}^{\infty} Y^t,$$

where we set $Y^0 \equiv \emptyset$. Each player has a private history: an agent's private history includes both the public history and his own actions in stages he was not monitored, $h_i^t \equiv (y^0, a_i^0; y^1, a_2^1; ...; y^{t-1}, a_i^{t-1})$. That is, the set of histories for Agent i, $i = 1, 2$, is

$$\mathscr{H}_i \equiv \cup_{t=0}^{\infty} (A_i \times Y)^t,$$

where we set $(A_i \times Y)^0 \equiv \emptyset$. The inspector's private history coincides with the public history.

A strategy of the inspector, σ_0, is a mapping from the set of all public histories into the set of mixed actions. Let ΔA_0 be the set of mixed actions for the inspector,

$$\sigma_0 : \mathscr{H} \to \Delta A_0.$$

Denote by $\mathscr{B}_0$ the set of all strategies of the inspector in the infinite repeated game.

It is assumed that the inspector announces his entire inspection strategy, $\sigma_0 \in \mathscr{B}_0$, at the beginning of the game and he is able to commit to it. We assume that the inspector has a *Stackelberg leader advantage*, because the inspector typically can publicly announce his inspection strategy.

For simplification, we focus on *pure public strategy* of the agents. An agent's pure public strategy is a pure strategy which depends only on the public history and *not* on the agent's private history. It is then a mapping from the set of inspection strategy $\mathscr{B}_0$ and all public histories into the set of pure actions A_i. That is, for $i = 1, 2$,

$$\sigma_i : \mathscr{B}_0 \times \mathscr{H} \to A_i.$$

We will concentrate on equilibria in which the agents play a pure public strategy and the inspector plays a behavior public strategy. In such an equilibrium, the agents cannot gain by deviating to a behavior public strategy. Since this is a game with perfect recall, a behavior public strategy is equivalent to a mixed public strategy (Kuhn's theorem). Therefore there is also no profitable deviation to a mixed public strategy. However, there might be additional equilibria in which *both* players play a behavior public strategy. We will not consider such equilibria.

Denote by $\sigma = (\sigma_0, \sigma_1, \sigma_2)$ the vector of the players' strategies. A *play* induced by σ is an infinite sequence of action profiles $\mathbf{a}(\sigma) \equiv (a^0(\sigma), a^1(\sigma), a^2(\sigma), ...)$, where $a^t(\sigma) = (a_i^t(\sigma))_{i \in N}$ is the action vector of the players in stage t. In stage t, the action profile $a^t(\sigma)$ yields a payoff of $u_i(a^t(\sigma))$ to player i. A play $\mathbf{a}(\sigma)$ thus implies an infinite stream of stage-game payoffs to each agent, given by $\big(u_i(a^0(\sigma)), u_i(a^1(\sigma)), u_i(a^2(\sigma)), ...\big) \in \mathbb{R}^\infty$. The collection of all the possible plays of the infinitely repeated game is denoted by $A^\infty = A^{\mathbb{N}}$. Every vector of strategies σ induces a probability distribution $\mathbf{P}_\sigma$ over the set A^∞. We denote by $\mathbf{E}_\sigma$ the expectation operator that corresponds to the probability distribution $\mathbf{P}_\sigma$; i.e., for every function $f : H^\infty \to \mathbb{R}$, the expectation of f under $\mathbf{P}_\sigma$ is denoted by $\mathbf{E}_\sigma[f]$:

$$\mathbf{E}_\sigma[f] = \int_{a \in A^\infty} f(a) d\mathbf{P}_\sigma(a).$$

Player i's expected payoff in stage t, under the strategy vector σ, is $\mathbf{E}_\sigma[u_i^t]$. Denote player

i's discounted payoff under strategy vector σ by

$$v_i(\sigma) := \mathbf{E}_\sigma \left[\sum_{t=1}^{\infty} \delta^{t-1} u_i^t \right]. \tag{5}$$

Note that the agents know their stage payoff while the inspector does not. Denote the subgame following the announcement of an inspection scheme σ_0 by $\Gamma(\sigma_0)$. Strategies (σ_1, σ_2) is a *Nash equilibrium* of $\Gamma(\sigma_0)$ if for every σ_1',

$$v_1(\sigma_0, \sigma_1, \sigma_2) \geq v_1(\sigma_0, \sigma_1', \sigma_2),$$

and for every σ_2',

$$v_2(\sigma_0, \sigma_1, \sigma_2) \geq v_2(\sigma_0, \sigma_1, \sigma_2').$$

For every public history $h^t \in \mathscr{H}$, we define the *continuation* game to be the infinitely repeated games that begins in period t, following history h^t. For every strategy profile σ, the inspector's *continuation inspection strategy induced by* h^t, denoted $\sigma_0|_{h^t}$, is given by

$$\sigma_0|_{h^t}(h^\tau) = \sigma_0(h^t h^\tau), \forall h^\tau \in \mathscr{H},$$

where $h^t h^\tau$ is the concatenation of the history h^t followed by the history h^τ. This is the inspection behavior implied by the strategy σ_0 in the continuation game that follows history h^t.

Agent i's *continuation strategy induced by* h^t, denoted $\sigma_i|_{h^t}$, is given by

$$\sigma_i|_{h^t}(\sigma_0|_{h^t}, h^\tau) = \sigma_i(\sigma_0, h^t h^\tau), \forall h^\tau \in \mathscr{H}.$$

Strategies (σ_1, σ_2) is a *subgame-perfect equilibrium* of $\Gamma(\sigma_0)$ if for all public histories $h^t \in \mathscr{H}$, $(\sigma_1|_{h^t}, \sigma_2|_{h^t})$ is a Nash equilibrium of $\Gamma(\sigma_0|_{h^t})$.

We focus only on the inspection strategies σ_0 such that in $\Gamma(\sigma_0)$ the set of pure subgame-perfect equilibria is nonempty. Denote by $\mathscr{E}$ the set of all such inspection strategies,[1] and for every $\sigma_0 \in \mathscr{E}$ denote by $(\sigma_1^{\sigma_0}, \sigma_2^{\sigma_0})$ the equilibrium strategy in $\Gamma(\sigma_0)$ that yields the inspector the highest expected payoff. Our goal is to find out the optimal inspection strategy σ_0^* of the inspector,

$$\sigma_0^* = \operatorname*{argmax}_{\sigma_0 \in \mathscr{E}} v_0(\sigma_0, \sigma_1^{\sigma_0}, \sigma_2^{\sigma_0}). \tag{6}$$

That is, to find the Nash equilibrium (where agents play public pure strategies and the

[1] It is easy to verify that $\mathscr{E}$ is non-empty. For instance, suppose for every $h^t \in \mathscr{H}$, $\hat{\sigma}_0(h^t) = I_1$, $\hat{\sigma}_1(h^t) = A$ and $\hat{\sigma}_2(h^t) = V$. Then $(\hat{\sigma}_1, \hat{\sigma}_2)$ is a subgame-perfect equilibrium of $\Gamma(\hat{\sigma}_0)$. Therefore $\hat{\sigma}_0 \in \mathscr{E}$.

inspector plays mixed strategies) of the Stackelberg game Γ.

Denote by $\sigma^* = (\sigma_0^*, \sigma_1^{\sigma_0^*}, \sigma_2^{\sigma_0^*})$ (one of) the Nash equilibrium strategy. Note first that

$$v_0(\sigma^*) \leq -\big(v_1(\sigma^*) + v_2(\sigma^*)\big). \tag{7}$$

This is because in each instance an agent chooses V, the “damage" he imposes on the inspector is at least as much as his benefit (see (1) - (3)).

Denote by $\sigma_0(a_0)$ the probability that the inspector plays action a_0 in the first stage, and by $\sigma_0(a_0|h^{t-1})$ the conditional probability that the inspector plays action a_0 in stage t, given that the public history in the first $t-1$ periods is h^{t-1}. Given every inspection strategy σ_0, for $i = 1, 2$ denote by $\sigma_i^{\sigma_0}(a_i)$ the probability that agent i plays action a_i in the first stage, and by $\sigma_i^{\sigma_0}(a_i|h^{t-1})$ the conditional probability that agent i plays action a_i in stage t, given the public history h^{t-1}. Note that since we consider only the pure public strategies of the agents, $\sigma_i^{\sigma_0}(a_i|h^{t-1})$ is either 1 or 0.

Proposition 1. $\max\big(v_1(\sigma^*), v_2(\sigma^*)\big) > 0.$

That is, *no* inspection strategy can deter both agents from violating, forever. By (7) and Proposition 1, $v_0(\sigma^*) < 0$.

Proof. Suppose $v_1(\sigma^*) = v_2(\sigma^*) = 0$. Without loss of generality let $\sigma_0^*(I_1) \leq \sigma_0^*(I_2)$. Since $\sigma_0^*(I_1) + \sigma_0^*(I_2) \leq 1$, $\sigma_0^*(I_1) \leq \frac{1}{2}$. If Agent 1 chooses V in the first period, he obtains

$$\big(1 - \sigma_0^*(I_1)\big) - c \cdot \sigma_0^*(I_1) + \delta \cdot v'.$$

Here v' is Agent 1's expected continuation payoff if he chooses V in period 1. Clearly $v' \geq 0$ since Agent 1 can guarantee 0 by choosing A in every period t, $t \geq 2$. Since $\sigma_0^*(I_1) \leq \frac{1}{2}$ and $c < 1$,

$$\big(1 - \sigma_0^*(I_1)\big) - c \cdot \sigma_0^*(I_1) + \delta \cdot v' \geq \big(1 - \sigma_0^*(I_1)\big) - c \cdot \sigma_0^*(I_1) > 0. \tag{8}$$

Therefore Agent 1 can obtain a positive payoff by choosing V in the first period, contradicting to $v_1(\sigma^*) = 0$. □

Assumption 1. $\sigma_i^{\sigma_0}(A|h^t) = 1$ *if* $v_i(\sigma|_{(h^t, V_i)}) = v_i(\sigma|_{(h^t, A_i)})$.

Assumption 1 asserts that in case of indifference, each agent prefers A to V. That is, an agent will *not* choose V unless it is beneficial - this reduces the number of equilibrium outcomes in $\Gamma(\sigma_0)$.

Proposition 2. *There exists an equilibrium σ^*, under which*

$$\sigma_i^{\sigma_0^*}(A|h^{t-1}) = 1 \quad \textit{iff} \quad v_i(\sigma^*|_{(h^{t-1},A_i)}) \geq f\big(\sigma_0^*(I_i|h^{t-1})\big), \tag{9}$$

where

$$f(p) = \begin{cases} \frac{1-p-cp}{p\delta} & \textit{if } 0 \leq p < \frac{1}{1+c} \\ 0 & \textit{if } \frac{1}{1+c} \leq p \leq 1. \end{cases} \tag{10}$$

Proof. See A.1 of the Appendix. □

Proposition 2 asserts that there exists an equilibrium under which each agent's V/A decision in period $t-1$ depends only on the probability that he will be inspected in the current period, as well as the (expected) continuation payoff if being (inspected and) found A. In this equilibrium, the inspector imposes the most severe punishment on the agent who deviates from A - to inspect on him with probability 1 in every continuation period.

By Proposition 2, if an agent is inspected with probability p in period $t-1$, he chooses A if and only if the expected continuation payoff he will obtain if he is (inspected and) found A is no less than $f(p)$. In particular, if $p \geq \frac{1}{1+c}$, this agent is best off choosing A in the current period, regardless of the continuation payoff. Note that an agent's expected continuation payoff if he is *not* inspected has *no* effect on his current-period action.

Consider an equilibrium σ^* and suppose that $v_0(\sigma^*) > \frac{1}{1-\delta}$.

By (7),

$$v_0(\sigma) \leq -\big(v_1(\sigma^*) + v_2(\sigma^*)\big). \tag{11}$$

We next show that there exists an equilibrium under which the equality in (11) holds.

Proposition 3. *Suppose $v_0(\sigma^*) > -\frac{1}{1-\delta}$. There exists an equilibrium σ^* under which*

$$v_0(\sigma^*) = -\big(v_1(\sigma^*) + v_2(\sigma^*)\big) \tag{12}$$

Proof. See A.2 of the Appendix. □

Lemma 1. *(i) For every $x \in \left[0, \frac{1}{1-\delta}\right]$ there exists a strategy $\bar{\sigma}_0$ of the inspector that yields agent i an expected payoff of x. (ii) Suppose σ^* is an equilibrium of Γ. Then $v_i(\sigma^*) \leq f\left(\frac{c}{1+c}\right)$ for $i = 1, 2$.*

Proof. See A.3 of the Appendix. □

Conditional on agent 1 obtains an expected payoff $x \geq 0$, we would like to find the inspection strategy of the inspector that minimizes the expected payoff of the other agent.

Denote by $g(x)$ the lowest equilibrium payoff of agent j when agent i, $i \neq j$, obtains an expected payoff of x.

Proposition 4. *(i) $g(x)$ is non-negative and non-increasing; (ii) $g(x)$ is convex; (iii) $g(0) = \frac{1-c^2}{1+c-\delta}$ and $g(\frac{1-c^2}{1+c-\delta}) = 0$; (iv) for $\forall x \in [0, \frac{1-c^2}{1+c-\delta}]$, $g\big(g(x)\big) = x$.*

Proof. See A.4 of the Appendix. □

Denote $w(p) = p \cdot f(p) + (1-p) \cdot g\big(f(1-p)\big)$. Note that $w(p)$ is strictly decreasing in p for $p \in [\frac{c}{1+c}, \frac{1}{1+c}]$.

Proposition 5. *For $\forall x \in (0, \frac{1-c^2}{1+c-\delta})$, let $p_1 = w^{-1}\left(\frac{x}{\delta}\right)$. $v_1(\sigma) = x$ and $v_2(\sigma) = g(x)$ can be implemented by the strategy σ_0: (i) $\sigma_0(\emptyset) = 0$, (ii) $v_1(\sigma|_{A_1}) = f(p_1)$, $v_2(\sigma|_{A_1}) = g\big(f(p_1)\big)$, $v_1(\sigma|_{A_2}) = g\big(f(1-p_1)\big)$ and $v_2(\sigma|_{A_2}) = f(1-p_1)$.*

Proof. See A.5 of the Appendix.

□

The next corollary follows immediately.

Corollary 1. *In equilibrium both agents choose Adhere before one agent's expected payoff reaches zero.*

After an agent's continuation payoff dropped down to zero, the other agent (who obtains a positive payoff) chooses Violate periodically - see A.3 of the Appendix for a detailed characterization.

Figure 8 follows immediately from Proposition 5.

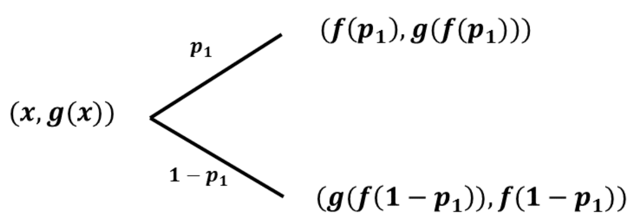

Figure 1

For every $x \in (0, \frac{1-c^2}{1+c-\delta})$,

$$x = 0 + \delta \cdot \Big(p_1 \cdot f(p_1) + (1-p_1) \cdot g\big(f(1-p_1)\big)\Big), \tag{13}$$

$$g(x) = 0 + \delta \cdot \Big(p_1 \cdot g\big(f(p_1)\big) + (1-p_1) \cdot f(1-p_1)\Big). \tag{14}$$

Since $p_1 = w^{-1}\left(\frac{x}{\delta}\right)$, (13) holds automatically. By (14),

$$g(x) = \delta \cdot w\left(1 - w^{-1}\left(\frac{x}{\delta}\right)\right). \tag{15}$$

In order to characterize (one of) the optimal inspection scheme, we need to find the function $g(\cdot)$ that satisfies (15) for every $x \in (0, \frac{1-c^2}{1+c-\delta})$. The analytical solution is difficult to solve, however, we can find the numerical solution for every given pairs of parameters (c, δ). See Section 3 for an example.

Corollary 2. *For $\forall x \in (0, \frac{1-c^2}{1+c-\delta})$, in the equilibrium that is characterized in Proposition 5: (i) $v_1(\sigma|_{A_1}) > x$, (ii) $v_1(\sigma|_{A_2}) < x$, (iii) $v_2(\sigma|_{A_1}) < g(x)$, (iv) $v_2(\sigma|_{A_2}) > g(x)$.*

Proof. See A.6 of the Appendix. □

Proposition 6. *In equilibrium (i) $v_1(\sigma) = v_2(\sigma) > 0$; (ii) $\sigma_0(I_1) = \sigma_0(I_2) = 0.5$.*

Proof. See A.7 of the Appendix. □

Proposition 6 asserts that, in the first period the inspector inspects on each agent with probability 0.5, and both agents obtain the same expected payoff.

Suppose the expected payoff of agent 1 and 2 in the current period is x and $g(x)$, respectively, where $x \in (0, \frac{1-c^2}{1+c-\delta})$. By Corollary 1 in equilibrium both agents will be inspected with positive probabilities and both will choose A in the current period. Depending on the realization of the principal's current-period inspection action, the agent that is inspected upon will obtain a higher expected payoff in the next period (compared with his current-period payoff), and the agent that is *not* inspected will obtain a lower expected payoff. W.l.o.g. suppose agent 1 is inspected in the current period. If $f(p_1) \in (0, \frac{1-c^2}{1+c-\delta})$, then $v_1(\sigma|_{A_1}) = f(p_1)$ and $v_2(\sigma|_{A_1}) = g\big(f(p_1)\big)$ can be implemented by the same strategy as setting $x = f(p_1)$. In the case where $f(p_1) \geq \frac{1-c^2}{1+c-\delta}$, $g\big(f(p_1)\big) = 0$ and this outcome can be implemented by the strategy described in A.4 (iii) of the Appendix.

3 Example

This section characterizes the optimal inspection scheme when $c = 0.7$ and $\delta = 0.9$. Figure 2 shows the the corresponding $g(\cdot)$ function.

One of the optimal inspection strategy is summarized in Figure 9), with the numbers on the lines representing the inspection probability on agent 1 and 2, and the numbers in the parentheses representing the expected continuation payoff of agent 1 and 2, respectively. In period 1, each agent's expected payoff is 0.18 and the inspector inspects on each agent with

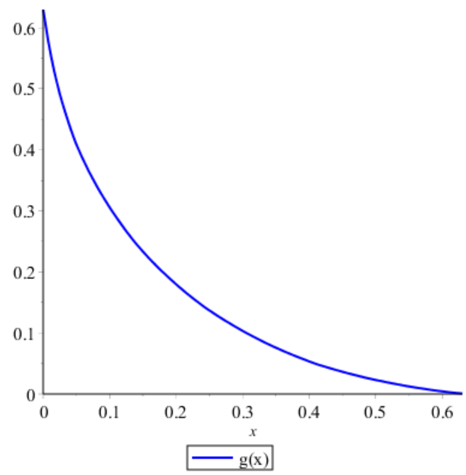

Figure 2: $g(x)$ for Case $c = 0.7$ and $\delta = 0.9$

probability 0.5. Suppose agent 1 is inspected in the first period, then in the second period, agent 1's expected payoff increases to 0.33 while agent 2's expected payoff drops to 0.08. Suppose, again agent 1 is inspected in the second period, then in the third period, agent 1's expected payoff increases again, to 0.54, and agent 2's payoff drops again, to 0.015. If agent 1 is inspected for another time in the third period, Phase 1 ends and agent 1 and 2 obtains the expected payoff of 0.75 and 0, respectively. If, however, in the third period, agent 2 is inspected, then in the forth period agent 2's expected payoff increases, and agent 1's payoff decreases. Similar process continues until one of the agent's payoff drops to 0.

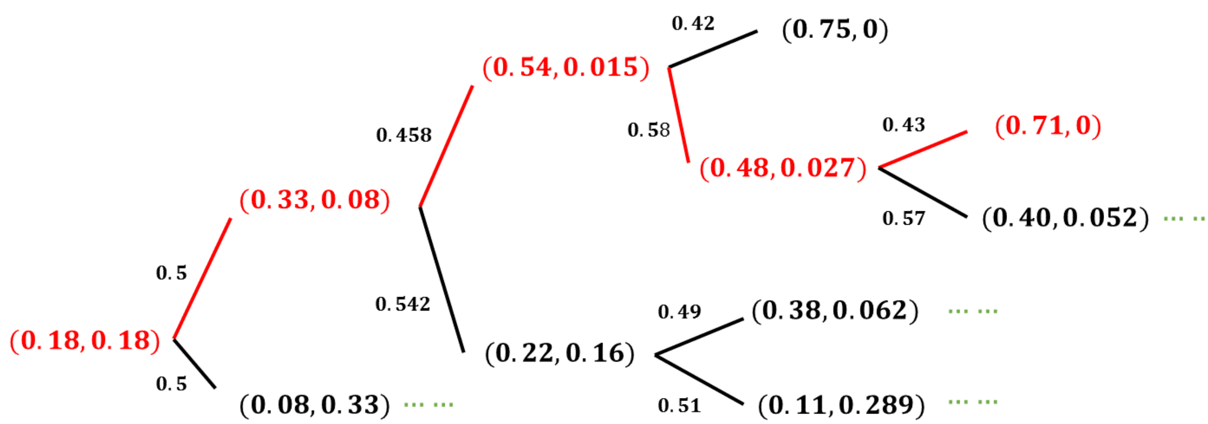

Figure 3: Phase 1 for Case $c = 0.7$ and $\delta = 0.9$

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A Appendix

A.1 Proof of Proposition 2

Lemma 2. $\sigma_i^{\sigma_0}(A|h^{t-1}) = 1$ *iff*

$$v_i(\sigma|_{(h^{t-1},A_i)}) - v_i(\sigma|_{(h^{t-1},V_i)}) \geq \frac{\left(1-\sigma_0(I_i|h^{t-1})\right)-c\cdot\sigma_0(I_i|h^{t-1})}{\sigma_0(I_i|h^{t-1})\cdot\delta}. \tag{16}$$

Proof. Without loss of generality let $i = 1$. Denote $p_1 = \sigma_0(I_1|h^{t-1})$. If Agent 1 chooses V, he obtains

$$A \equiv (1 - p_1) - c \cdot p_1 + (1 - p_1)\delta v^{NI} + p_1 \delta v^V, \tag{17}$$

where v^{NI} is Agent 1's expected continuation payoff if he is not being inspected in period t, and v^V is his expected continuation payoff if he is being inspected in period t and found V.

If Agent 1 chooses A, he obtains

$$B \equiv 0 + (1 - p_1)\delta v^{NI} + p_1 \delta v^A, \tag{18}$$

where v^A is Agent 1's expected continuation payoff if he is inspected in period t and found A. It worth notice that Agent 1's expected continuation payoff if he is *not* inspected is the same regardless of his action in period t.

$$B - A = p_1\delta\big(v^A - v^V\big) - \big((1 - p_1) - cp_1\big) \geq 0,$$

and Lemma 2 follows. $\square$

By Lemma 2, agent i's V/A decision in period t (following the history h^{t-1}) depends only on the probability that he will be inspected in the current period; the (expected) continuation payoff if being (inspected and) found V, as well as his payoff if being (inspected and) found A. In particular, it does *not* depend on agent i's (expected) continuation payoff if he is *not* inspected.

Lemma 3. *There exists an optimal strategy of the inspector, σ_0, under which $v_i(\sigma|_{(h^t,V_i)}) \leq v_i(\sigma|_{(h^t,A_i)})$ for $i = 1, 2$ and for every $h^t \in \mathscr{H}$.*

Note that $\sigma = (\sigma_0, \sigma_1^{\sigma_0}, \sigma_2^{\sigma_0})$. Lemma (3) asserts that there exists an optimal inspection strategies under which the inspector does *not* reward violating. That is, an agent's expected continuation payoff if he is (inspected and) found V is no more than his expected continuation payoff if he is (inspected and) found A.

Proof. Suppose there exists an equilibrium $\hat{\sigma} = (\hat{\sigma}_0, \sigma_1^{\hat{\sigma}_0}, \sigma_2^{\hat{\sigma}_0})$ under which there exists a history $\hat{h}^{k-1}$, $v_1(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}) > v_1(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)})$.

Case 1: suppose

$$v_1(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)}) - v_1(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}) \geq \frac{\left(1-\hat{\sigma}_0(I_1|\hat{h}^{k-1})\right)-c\cdot\hat{\sigma}_0(I_1|\hat{h}^{k-1})}{\hat{\sigma}_0(I_1|\hat{h}^{k-1})\cdot\delta}. \tag{19}$$

By Lemma 2, $\sigma_1^{\hat{\sigma}_0}(A|\hat{h}^{k-1}) = 1$. In this case consider the inspection strategy $\tilde{\sigma}_0$, which is the same as $\hat{\sigma}_0$ except that $\tilde{\sigma}_0|_{(\hat{h}^{k-1},V_1)} = \hat{\sigma}_0|_{(\hat{h}^{k-1},A_1)}$. It can be easily verified that

$$\begin{aligned} v_1(\tilde{\sigma}|_{(\hat{h}^{k-1},V_1)}) &= v_1(\tilde{\sigma}|_{(\hat{h}^{k-1},A_1)}) \\ &= v_1(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)}) < v_1(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}). \end{aligned} \tag{20}$$

Since $\tilde{\sigma}_0(I_1|\hat{h}^{k-1}) = \hat{\sigma}_0(I_1|\hat{h}^{k-1})$,

$$v_1(\tilde{\sigma}|_{(\hat{h}^{k-1},A_1)}) - v_1(\tilde{\sigma}|_{(\hat{h}^{k-1},V_1)}) \geq \frac{\left(1-\tilde{\sigma}_0(I_1|\hat{h}^{k-1})\right)-c\cdot\tilde{\sigma}_0(I_1|\hat{h}^{k-1})}{\tilde{\sigma}_0(I_1|\hat{h}^{k-1})\cdot\delta} \tag{21}$$

and $\sigma_1^{\tilde{\sigma}_0}(A|\hat{h}^{k-1}) = 1$. It is then easy to verify that $\tilde{\sigma} = (\tilde{\sigma}_0, \sigma_1^{\tilde{\sigma}}, \sigma_2^{\tilde{\sigma}})$ is also an equilibrium of Γ.

Case 2: suppose

$$v_1(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)}) - v_1(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}) < \frac{\left(1-\hat{\sigma}_0(I_1|\hat{h}^{k-1})\right)-c\cdot\hat{\sigma}_0(I_1|\hat{h}^{k-1})}{\hat{\sigma}_0(I_1|\hat{h}^{k-1})\cdot\delta}. \tag{22}$$

By Lemma 2, $\sigma_1^{\hat{\sigma}_0}(V|\hat{h}^{k-1}) = 1$.

We construct another strategy of the inspector, σ_0', which satisfies (i) $\sigma_0'(a_0|h) = \hat{\sigma}_0(a_0|h)$ for $\forall h \in \cup_{t=0}^{k-1} Y^t$. (ii) $\sigma_0'|_{(h^{k-1},y)} = \hat{\sigma}_0|_{(h^{k-1},y)}$ for every $h^{k-1} \neq \hat{h}^{k-1}$. (iii) $\sigma_0'|_{(\hat{h}^{k-1},y)} = \hat{\sigma}_0|_{(\hat{h}^{k-1},y)}$ for every $y \neq A_1$. (iv) $\sigma_0'|_{(\hat{h}^{k-1},A_1)} = \hat{\sigma}_0|_{(\hat{h}^{k-1},V_1)}$. Namely, σ_0' is similar to $\hat{\sigma}_0$ except that following history $\hat{h}^{k-1}$, if agent 1 in period $k-1$ is inspected and found A, the inspector acts as if agent 1 is found V. We claim that $\sigma' = (\sigma_0', \sigma_1^{\sigma_0'}, \sigma_2^{\sigma_0'})$ is an equilibrium of Γ.

Now we consider agents' equilibrium strategy under σ_0'. By (ii) and (iii), for $i = 1, 2$, $\sigma_i^{\sigma_0'}|_{(h^{k-1},y)} = \sigma_i^{\hat{\sigma}_0}|_{(h^{k-1},y)}$ for either $h^{k-1} \neq \hat{h}^{k-1}$ or $y \neq V_1$. Therefore for $i = 1, 2$, $v_i\big(\sigma'|_{(h^{k-1},y)}\big) = v_i\big(\hat{\sigma}|_{(h^{k-1},y)}\big)$ for either $h^{k-1} \neq \hat{h}^{k-1}$ or $y \neq V_1$. By (iv), $\sigma_1^{\sigma_0'}|_{(\hat{h}^{k-1},A_1)}(h^\tau) = \sigma_1^{\hat{\sigma}_0}|_{(\hat{h}^{k-1},V_1)}(h^\tau)$ and $\sigma_2^{\sigma_0'}|_{(\hat{h}^{k-1},A_1)}(h^\tau) = \sigma_2^{\hat{\sigma}_0}|_{(\hat{h}^{k-1},V_1)}(h^\tau)$ for $\forall h^\tau \in \mathscr{H}$, thus $v_1\big(\sigma'|_{(\hat{h}^{k-1},A_1)}\big) = v_1\big(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\big)$ and $v_2\big(\sigma'|_{(\hat{h}^{k-1},A_1)}\big) = v_2\big(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\big)$. Since $v_1(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}) > v_1(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)})$ (by assumption), $v_1(\sigma'|_{(\hat{h}^{k-1},A_1)}) > v_1(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)})$.

We next consider agent i's equilibrium strategy at period $k-1$. By Lemma 2 agent 2's action following history $\hat{h}^{k-1}$ is the same in σ_0' and $\hat{\sigma}_0$. As for agent 1, there are two possibilities.

Subcase 2.1: suppose

$$v_1(\sigma'|_{(\hat{h}^{k-1},A_1)}) - v_1(\sigma'|_{(\hat{h}^{k-1},V_1)}) < \frac{\left(1-\sigma_0'(I_1|\hat{h}^{k-1})\right)-c\cdot\sigma_0'(I_1|\hat{h}^{k-1})}{\sigma_0'(I_1|\hat{h}^{k-1})\cdot\delta}, \tag{23}$$

then $\sigma_1^{\sigma_0'}(V|\hat{h}^{k-1}) = 1$. That is, the equilibrium path under $\hat{\sigma}$ and σ' is the same and $v_0(\sigma') = v_0(\hat{\sigma})$. It is then easy to verify that $\tilde{\sigma} = (\tilde{\sigma}_0, \sigma_1^{\tilde{\sigma}}, \sigma_2^{\tilde{\sigma}})$ is an equilibrium of Γ.

Subcase 2.2: suppose

$$v_1(\sigma'|_{(\hat{h}^{k-1},A_1)}) - v_1(\sigma'|_{(\hat{h}^{k-1},V_1)}) \geq \frac{\left(1-\sigma_0'(I_1|\hat{h}^{k-1})\right)-c\cdot\sigma_0'(I_1|\hat{h}^{k-1})}{\sigma_0'(I_1|\hat{h}^{k-1})\cdot\delta}, \tag{24}$$

then $\sigma_1^{\sigma_0'}(A|\hat{h}^{k-1}) = 1$. Since $v_1\left(\sigma'|_{(\hat{h}^{k-1},A_1)}\right) = v_1\left(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\right)$ and $v_2\left(\sigma'|_{(\hat{h}^{k-1},A_1)}\right) = v_2\left(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\right)$, $v_1\left(\sigma'|_{\hat{h}^{k-1}}\right) = v_1\left(\hat{\sigma}|_{\hat{h}^{k-1}}\right)$ and $v_2\left(\sigma'|_{\hat{h}^{k-1}}\right) = v_2\left(\hat{\sigma}|_{\hat{h}^{k-1}}\right)$. Therefore the choice of agent 1 and 2 following history $\hat{h}^{k-2}$ (here $\hat{h}^{k-2}$ is the history that is consistent with the first $k-2$ periods of $\hat{k}^{k-1}$) is the same under $\hat{\sigma}_0$ and σ'. The only difference between the the equilibrium path induced by these two inspection strategies is that following $\hat{h}^{k-1}$, agent 1 chooses V under $\hat{\sigma}_0$; while he chooses A under σ_0'. The inspector's payoff under σ' is then no less than his payoff under $\hat{\sigma}$. Namely, σ' is also an equilibrium of Γ.

Since the same argument can be made for each history h^t under which $v_1(\hat{\sigma}|_{(h^t,V_1)}) > v_1(\hat{\sigma}|_{(h^t,A_1)})$, Lemma 3 follows.

$\square$

Lemma 4. *(i) If $\sigma_0(I_i|h^{t-1}) \geq \frac{1}{1+c}$, then $\sigma_i^{\sigma_0}(A|h^{t-1}) = 1$. (ii) If $\sigma_0(I_i|h^{t-1}) < \frac{1}{1+c}$, then $\sigma_i^{\sigma_0}(A|h^{t-1}) = 1$ iff*

$$v_i(\sigma|_{(h^{t-1},A_i)}) - v_i(\sigma|_{(h^{t-1},V_i)}) \geq \frac{\left(1-\sigma_0(I_i|h^{t-1})\right)-c\cdot\sigma_0(I_i|h^{t-1})}{\sigma_0(I_i|h^{t-1})\cdot\delta}. \tag{25}$$

Lemma 4 states that if the probability of an agent being inspected in period t is no less than $\frac{1}{1+c}$, this agent is best off choosing A in that period. If the probability of an agent being inspected in period t is less than $\frac{1}{1+c}$, this agent chooses A if and only if the "reward" he will obtain if he is found A is sufficiently higher than the "punishment" he faces if he is found V.

Proof. By Lemma 3, $v_i(\sigma|_{(h^{t-1},A_i)}) \geq v_i(\sigma|_{(h^{t-1},V_i)})$. Lemma 4 follows immediately from Lemma 2. $\square$

Lemma 5. *There exists an optimal strategy of the inspector, σ_0^*, such that for every history h^{t-1} satisfying $\sigma_i^{\sigma_0^*}(A|h^{t-1}) = 1$, $v_i(\sigma^*|_{(h^{t-1},V_i)}) = 0$.*

Note that $\sigma^* = (\sigma_0^*, \sigma_1^{\sigma_0^*}, \sigma_2^{\sigma_0^*})$. Lemma 5 asserts that the inspector cannot loss by imposing the most severe punishment on an agent if this agent deviates from A. Since every equilibrium inspection strategy yields the inspector the same payoff, we next focus only on those with the structure described in Lemma 5.

Proof. Suppose there exists an equilibrium $\hat{\sigma} = (\hat{\sigma}_0, \sigma_1^{\hat{\sigma}_0}, \sigma_2^{\hat{\sigma}_0})$ under which there exists a history $\hat{h}^{k-1}$, $\sigma_1^{\hat{\sigma}_0}(A|\hat{h}^{k-1}) = 1$, while $v_1\big(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\big) > 0$.

We construct another strategy of the inspector, σ_0', which satisfies (i) $\sigma_0'(a_0|h) = \hat{\sigma}_0(a_0|h)$ for $\forall h \in \cup_{t=0}^{k-1} Y^t$. (ii) $\sigma_0'|_{(h^{k-1},y)} = \hat{\sigma}_0|_{(h^{k-1},y)}$ for every $h^{k-1} \neq \hat{h}^{k-1}$. (iii) $\sigma_0'|_{(\hat{h}^{k-1},y)} = \hat{\sigma}_0|_{(\hat{h}^{k-1},y)}$ for every $y \neq V_1$. (iv) $\sigma_0'(I_1|(\hat{h}^{k-1}, V_1, h^\tau)) = 1$ for $\forall h^\tau \in \mathscr{H}$. Namely, σ_0' is similar to $\hat{\sigma}_0$ except that following history $\hat{h}^{k-1}$, if agent 1 in period $k-1$ is inspected and found V, the inspector will inspect on him with probability 1 for every following period.

Now we consider agents' equilibrium strategy under σ_0'. By (ii) and (iii), for $i = 1,2$, $\sigma_i^{\sigma_0'}|_{(h^{k-1},y)} = \sigma_i^{\hat{\sigma}_0}|_{(h^{k-1},y)}$ for either $h^{k-1} \neq \hat{h}^{k-1}$ or $y \neq V_1$. Therefore for $i = 1,2$, $v_i\big(\sigma'|_{(h^{k-1},y)}\big) = v_i\big(\hat{\sigma}|_{(h^{k-1},y)}\big)$ for either $h^{k-1} \neq \hat{h}^{k-1}$ or $y \neq V_1$. By (iv), $\sigma_1^{\sigma_0'}|_{(\hat{h}^{k-1},V_1)}(h^\tau) = A$ and $\sigma_2^{\sigma_0'}|_{(\hat{h}^{k-1},V_1)}(h^\tau) = V$ for $\forall h^\tau \in \mathscr{H}$, thus $v_1\big(\sigma'|_{(\hat{h}^{k-1},V_1)}\big) = 0$ and $v_2\big(\sigma'|_{(\hat{h}^{k-1},V_1)}\big) = \frac{1}{1-\delta}$.

We next consider agents' equilibrium strategy at period $k-1$. Note that under σ_0, each agent's V/A decision in period $k-1$ depends only on $v_i(\sigma_0|_{(h^{k-1},A_i)})$, $v_i(\sigma_0|_{(h^{k-1},V_i)})$ and $\sigma_0(I_i|h^{k-1})$, but *not* on the other agent's action in the current period (see Lemma 4). Thus by (i) and (ii), for every $h^{k-1} \neq \hat{h}^{k-1}$, $\sigma_i^{\sigma_0'}(a_i|h^{k-1}) = \sigma_i^{\hat{\sigma}_0}(a_i|h^{k-1})$ for every $a_i \in A_i$. Moreover, $\sigma_2^{\sigma_0'}(a_2|\hat{h}^{k-1}) = \sigma_2^{\hat{\sigma}_0}(a_2|\hat{h}^{k-1})$ for every $a_2 \in A_2$. It is left to analyze agent 1's equilibrium strategy under history $\hat{h}^{k-1}$. Here we need to distinguish between two cases. In case $\hat{\sigma}_0(I_1|\hat{h}^{k-1}) \geq \frac{1}{1+c}$, $\sigma_1^{\sigma_0'}(A|\hat{h}^{k-1}) = \sigma_1^{\hat{\sigma}_0}(A|\hat{h}^{k-1}) = 1$ regardless of the value of $v_1\big(\sigma'|_{(\hat{h}^{k-1},V_1)}\big)$ and $v_1\big(\sigma'|_{(\hat{h}^{k-1},A_1)}\big)$, as long as $v_1\big(\sigma'|_{(\hat{h}^{k-1},A_1)}\big) \geq v_1\big(\sigma'|_{(\hat{h}^{k-1},V_1)}\big)$ (see Lemma 4). In case $\hat{\sigma}_0(I_1|\hat{h}^{k-1}) < \frac{1}{1+c}$, $\sigma_1^{\hat{\sigma}_0}(A|\hat{h}^{k-1}) = 1$ implies that

$$v_1\big(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)}\big) - v_1\big(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\big) \geq \frac{\big(1-\hat{\sigma}_0(I_1|\hat{h}^{k-1})\big) - c\cdot\hat{\sigma}_0(I_1|\hat{h}^{k-1})}{\hat{\sigma}_0(I_1|\hat{h}^{k-1})\cdot\delta}.$$

Since $v_1\big(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}\big) > 0$,

$$v_1\big(\hat{\sigma}|_{(\hat{h}^{k-1},A_1)}\big) > \frac{\big(1-\hat{\sigma}_0(I_1|\hat{h}^{k-1})\big) - c\cdot\hat{\sigma}_0(I_1|\hat{h}^{k-1})}{\hat{\sigma}_0(I_1|\hat{h}^{k-1})\cdot\delta}.$$

This implies that

$$v_1\big(\sigma'|_{(\hat{h}^{k-1},A_1)}\big) - \overbrace{v_i\big(\sigma'|_{(\hat{h}^{k-1},V_1)}\big)}^{0} \geq \frac{\big(1-\sigma_0'(I_1|\hat{h}^{k-1})\big) - c\cdot\sigma_0'(I_1|\hat{h}^{k-1})}{\sigma_0'(I_1|\hat{h}^{k-1})\cdot\delta}.$$

Therefore $\sigma_1^{\sigma_0'}(A|\hat{h}^{k-1}) = \sigma_1^{\hat{\sigma}_0}(A|\hat{h}^{k-1}) = 1$. To summarize, for $i = 1, 2$, $\sigma_i^{\sigma_0'}(a_i|h^{k-1}) = \sigma_i^{\hat{\sigma}_0}(a_i|h^{k-1})$ for every $h^{k-1} \in Y^{k-1}$. Thus, $v_i(\sigma'|_{h^{k-1}}) = v_i(\hat{\sigma}|_{h^{k-1}})$. Combining this result with (i), $\sigma_i^{\sigma_0'}(a_i|h^{k-2}) = \sigma_i^{\hat{\sigma}_0}(a_i|h^{k-2})$ and $v_i(\sigma'|_{h^{k-2}}) = v_i(\hat{\sigma}|_{h^{k-2}})$ for every $h^{k-2} \in Y^{k-2}$. By induction, $\sigma_i^{\sigma_0'}(a_i|h) = \sigma_i^{\hat{\sigma}_0}(a_i|h)$ for every $h \in \cup_{t=0}^{k-1} Y^t$. This implies that $\sigma' = (\sigma_0', \sigma_1^{\sigma_0'}, \sigma_2^{\sigma_0'})$ and $\hat{\sigma} = (\hat{\sigma}_0, \sigma_1^{\hat{\sigma}_0}, \sigma_2^{\hat{\sigma}_0})$ induces the same probability distribution over the set of plays in the first $k-1$ periods. Since $\sigma'|_{(\hat{h}^{k-1}, y)} = \hat{\sigma}|_{(\hat{h}^{k-1}, y)}$ for $\forall y \neq V_1$ and since the history $(\hat{h}^{k-1}, V_1)$ occurs with probability 0 in both σ' and $\hat{\sigma}$, σ' induces the same probability distribution over the set of plays as σ'. By (5), $v_0(\sigma') = v_0(\hat{\sigma})$ and σ' is an equilibrium of Γ.

$\square$

Proposition 2 then follows immediately from Lemma 4 and 5.

A.2 Proof of Proposition 3

Rewrite

Lemma 6. *(i) $v_0(\sigma^*) \geq -\frac{1}{1-\delta}$. (ii) $v_0(\sigma^*) > -\frac{1}{1-\delta}$ if and only if there exists h^{t-1} under which $\sigma_1^{\sigma_0^*}(A|h^{t-1}) = \sigma_2^{\sigma_0^*}(A|h^{t-1}) = 1$.*

Proof. (i) Consider the following strategy of the inspector: $\tilde{\sigma}_0(I_1|h^{t-1}) = \frac{1}{1+c}$ and $\tilde{\sigma}_0(I_2|h^{t-1}) = \frac{c}{1+c}$ for every h^{t-1}. The best responds of the two agents are $\sigma_1^{\tilde{\sigma}_0}(A|h^{t-1}) = 1$ and $\sigma_2^{\tilde{\sigma}_0}(A|h^{t-1}) = 0$. The inspector obtains $v_0(\tilde{\sigma}_0, \sigma_1^{\tilde{\sigma}_0}, \sigma_2^{\tilde{\sigma}_0}) = -1 - \delta - \delta^2 - ... = -\frac{1}{1-\delta}$. By (6), $v_p(\sigma^*) \geq v_0(\tilde{\sigma}_0, \sigma_1^{\tilde{\sigma}_0}, \sigma_2^{\tilde{\sigma}_0}) \geq -\frac{1}{1-\delta}$. (ii) For the inspector to obtain an expected payoff higher than $-\frac{1}{1-\delta}$, it must be the case that in at least one stage both agents choose A. $\square$

Suppose $v_0(\sigma^*) > -\frac{1}{1-\delta}$ but in the first period $\sigma_1^{\sigma_0^*}(A) = 1$ and $\sigma_1^{\sigma_0^*}(V) = 0$. The inspector obtains -1 in the first period. From period 2, the inspector obtains

$$\delta \cdot \big(\overbrace{\sigma_0^*(I_1) \cdot v_0(\sigma^*|_{A_1}) + \sigma_0^*(I_2) \cdot v_0(\sigma^*|_{V_2}) + \sigma_0^*(\emptyset) \cdot v_0(\sigma^*|_{\emptyset})}^{v_0^2} \big),$$

where $v_0^2 > -\frac{1}{1-\delta}$. Now consider an alternative strategy σ_0', of the inspector. At the beginning of the first period the inspector uses a random device to determine the inspection strategy: with probability $\sigma_0^*(I_1)$ he chooses $\sigma_0^*|_{A_1}$; with probability $\sigma_0^*(I_2)$ he chooses $\sigma_0^*|_{V_2}$; and with probability $\sigma_0^*(\emptyset)$ he chooses $\sigma_0^*|_{\emptyset}$. Under this new inspection strategy the inspector obtains

$$v_0(\sigma') = v_0^2 > -1 + \delta \cdot v_0^2,$$

contradicts to the assumption that σ_0^* is optimal.

Lemma 7. *Suppose $v_0(\sigma^*) > -\frac{1}{1-\delta}$. There exists a public correlated equilibrium $\sigma^* = (\sigma_0^*, \sigma_1^{\sigma_0^*}, \sigma_2^{\sigma_0^*})$ under which*

$$\begin{aligned} v_0(\sigma^*) = -\delta\cdot\Big[&\sigma_0^*(I_1)\cdot\big(v_1(\sigma^*|_{A_1}) + v_2(\sigma^*|_{A_1})\big) + \sigma_0^*(I_2)\cdot\big(v_1(\sigma^*|_{A_2}) + v_2(\sigma^*|_{A_2})\big) \\ &+ \sigma_0^*(\emptyset)\cdot\big(v_1(\sigma^*|_{\emptyset}) + v_2(\sigma^*|_{\emptyset})\big)\Big] \end{aligned} \tag{26}$$

where $v_1(\sigma^*|_{A_1}) \geq f(\sigma_0^*(I_1))$ and $v_2(\sigma^*|_{A_2}) \geq f(\sigma_0^*(I_2))$.

Proof. The requirement that $v_1(\sigma^*|_{A_1}) \geq f(\sigma_0^*(I_1))$ and $v_2(\sigma^*|_{A_2}) \geq f(\sigma_0^*(I_2))$ follows immediately from Lemma 6 and Proposition 2. We next prove that there exists an equilibrium satisfying equality (26).

Suppose $\hat{\sigma} = (\hat{\sigma}_0, \sigma_1^{\hat{\sigma}_0}, \sigma_2^{\hat{\sigma}_0})$ is an equilibrium under which

$$v_0(\hat{\sigma}) < -\Big[\hat{\sigma}_0(I_1)\delta\cdot\big(v_1(\hat{\sigma}|_{A_1}) + v_2(\hat{\sigma}|_{A_1})\big) + \hat{\sigma}_0(I_2)\delta\cdot\big(v_1(\hat{\sigma}|_{A_2}) + v_2(\hat{\sigma}|_{A_2})\big)\Big]. \tag{27}$$

This implies that the total "damage" of V on the inspector exceeds the total benefits of agents. By (1) - (3), this can happen only if under a history on the equilibrium path, an agent chooses V while he is inspected with positive probability. That is, there exists a history $\hat{h}^{k-1}$ which occurs with positive probability in equilibrium, under which $\hat{\sigma}_0(I_i|\hat{h}^{k-1}) > 0$ and $\sigma_i^{\hat{\sigma}_0}(V|\hat{h}^{k-1}) = 1$ for some i. Without loss of generality let $i = 1$.

Now consider another inspection strategy σ_0'. (i) $\sigma_0'(a_0|h) = \hat{\sigma}_0(a_0|h)$ for $\forall h \in \cup_{t=0}^{k-1} Y^t$. (ii) $\sigma_0'(I_1|\hat{h}^{k-1}) = 0$, $\sigma_0'(\emptyset|\hat{h}^{k-1}) = \hat{\sigma}_0(\emptyset|\hat{h}^{k-1}) + \hat{\sigma}_0(I_1|\hat{h}^{k-1})$ and $\sigma_0'(I_2|\hat{h}^{k-1}) = \hat{\sigma}_0(I_2|\hat{h}^{k-1})$. (iii) $\sigma_0'|_{(\hat{h}^{k-1},\emptyset)} = \frac{\hat{\sigma}_0(\emptyset|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}\cdot\hat{\sigma}_0|_{(\hat{h}^{k-1},\emptyset)} + \frac{\hat{\sigma}_0(I_1|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}\cdot\hat{\sigma}_0|_{(\hat{h}^{k-1},V_1)}$ and $\sigma_0'|_{(\hat{h}^{k-1},y)} = \hat{\sigma}_0|_{(\hat{h}^{k-1},y)}$ for $y \neq \emptyset$. (iv) $\sigma_0'|_{(h^{k-1},y)} = \hat{\sigma}_0|_{(h^{k-1},y)}$ for every $h^{k-1} \neq \hat{h}^{k-1}$ and $y \in Y$.

Unlike $\hat{\sigma}_0$, in σ_0' under history $\hat{h}^{k-1}$, the inspector does *not* inspect on Agent 1. Under the history $(\hat{h}^{k-1},\emptyset)$, in σ_0' the inspector uses a correlation device: he plays $\hat{\sigma}_0|_{(\hat{h}^{k-1},\emptyset)}$ with probability $\frac{\hat{\sigma}_0(\emptyset|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}$ and plays $\hat{\sigma}_0|_{(\hat{h}^{k-1},V_1)}$ with the remaining probability.

We next analyze agents' equilibrium strategies under σ_0'. Whenever the inspector uses $\hat{\sigma}_0|_{(\hat{h}^{k-1},\emptyset)}$ (or $\hat{\sigma}_0|_{(\hat{h}^{k-1},V_1)}$), agent i's equilibrium strategy is $\sigma_i^{\hat{\sigma}_0}|_{(\hat{h}^{k-1},\emptyset)}$ (or $\sigma_i^{\hat{\sigma}_0}|_{(\hat{h}^{k-1},V_1)}$) and the expected payoff is $v_i(\hat{\sigma}|_{(\hat{h}^{k-1},\emptyset)})$ (or $v_i(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)})$). Thus for $i = 0, 1, 2$,

$$v_i(\sigma'|_{(\hat{h}^{k-1},\emptyset)}) = \frac{\hat{\sigma}_0(\emptyset|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}\cdot v_i(\hat{\sigma}|_{(\hat{h}^{k-1},\emptyset)}) + \frac{\hat{\sigma}_0(I_1|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}\cdot v_i(\hat{\sigma}|_{(\hat{h}^{k-1},V_1)}).$$

Since $\sigma_0'(I_1|\hat{h}^{k-1}) = 0$, $\sigma_1^{\sigma_0'}(V|\hat{h}^{k-1}) = \sigma_1^{\hat{\sigma}_0}(V|\hat{h}^{k-1}) = 1$. That is, agent 1 chooses V in period $k-1$, following the history $\hat{h}^{k-1}$. Since Agent 1's expected continuation payoff under

σ' and $\hat{\sigma}$ are the same, namely

$$\sum_{y\in Y}\sigma_0'(y|\hat{h}^{k-1})v_1(\sigma'|_{(\hat{h}^{k-1},y)})=\sum_{y\in Y}\hat{\sigma}_0(y|\hat{h}^{k-1})v_1(\hat{\sigma}|_{(\hat{h}^{k-1},y)}),$$

and Agent 1 under σ_0' is inspected with a lower probability in the current period (when he chooses V), $v_1(\sigma'|_{\hat{h}^{k-1}})>v_1(\hat{\sigma}|_{\hat{h}^{k-1}})$. As for Agent 2, his action in period $k-1$ depends only on the probability that he will be inspected in the current period, as well as the expected continuation payoff if he is inspected and found A. Since these two values are the same under σ' and $\hat{\sigma}$, $\sigma_2^{\sigma_0'}(a_2|\hat{h}^{k-1})=\sigma_2^{\hat{\sigma}_0}(a_2|\hat{h}^{k-1})$ for every $a_2\in A_2$. It is then easy to verify that $v_2(\sigma'|_{\hat{h}^{k-1}})=v_2(\hat{\sigma}|_{\hat{h}^{k-1}})$ and $v_0(\sigma'|_{\hat{h}^{k-1}})=v_0(\hat{\sigma}|_{\hat{h}^{k-1}})$ (by (2) - (3)).

To summarize,

$$v_i(\sigma'|_{\hat{h}^{k-1}})\geq v_i(\hat{\sigma}|_{\hat{h}^{k-1}}). \tag{28}$$

In particular,

$$v_0(\sigma'|_{\hat{h}^{k-1}})=v_0(\hat{\sigma}|_{\hat{h}^{k-1}}),v_2(\sigma'|_{\hat{h}^{k-1}})=v_2(\hat{\sigma}|_{\hat{h}^{k-1}})\text{ and }v_1(\sigma'|_{\hat{h}^{k-1}})>v_1(\hat{\sigma}|_{\hat{h}^{k-1}}). \tag{29}$$

Note that,

$$v_i(\sigma'|_{h^{k-1}})=v_i(\hat{\sigma}|_{h^{k-1}})\quad\text{for every}\quad h^{k-1}\neq\hat{h}^{k-1}. \tag{30}$$

Since history $\hat{h}^{k-1}$ occurs with positive probability, under history $\hat{h}^{k-2}$ which is consistent with $\hat{h}^{k-1}$, $v_i(\hat{\sigma}|_{\hat{h}^{k-1}})\geq v_i(\hat{\sigma}|_{h^{k-1}})$, for every $h^{k-1}\neq\hat{h}^{k-1}$. This implies that $v_i(\sigma'|_{\hat{h}^{k-1}})\geq v_i(\sigma'|_{h^{k-1}})$ for every $h^{k-1}\neq\hat{h}^{k-1}$ (by (28) and (30)). Thus, by (i), at period $k-1$, $\sigma_i^{\sigma_0'}(a_i|h^{k-2})=\sigma_i^{\hat{\sigma}_0}(a_i|h^{k-2})$[2]. By deduction, in every period $0\leq t\leq k-1$, $\sigma_i^{\sigma_0'}(a_i|h^t)=\sigma_i^{\hat{\sigma}_0}(a_i|h^t)$.

This implies that $\sigma'=(\sigma_0',\sigma_1^{\sigma_0'},\sigma_2^{\sigma_0'})$ and $\hat{\sigma}=(\hat{\sigma}_0,\sigma_1^{\hat{\sigma}_0},\sigma_2^{\hat{\sigma}_0})$ induces the same probability distribution over the set of plays in the first $k-1$ periods. Since $v_0(\sigma'|_{\hat{h}^{k-1}})=v_0(\hat{\sigma}|_{\hat{h}^{k-1}})$ (by 29), the principal obtains the same expected payoff in σ' and $\hat{\sigma}$. Therefore $(\sigma_0',\sigma_1^{\sigma_0'},\sigma_2^{\sigma_0'})$ is an equilibrium of Γ. Under σ_0, unlike in $\hat{\sigma}_0$, $\sigma_1^{\sigma_0'}(V|\hat{h}^{k-1})=1$ and $\sigma_0'(I_1|\hat{h}^{k-1})=0$. By the same reason we can construct an equilibrium $\sigma^*=(\sigma_0^*,\sigma_1^{\sigma_0^*},\sigma_2^{\sigma_0^*})$ under which for each instance $\sigma_1^{\sigma_0^*}(V|h^{t-1})=1$, $\sigma_0^*(I_1|h^{t-1})=0$ holds. Lemma 7 follows.

□

[2] Even in case $v_i(\hat{\sigma}|_{\hat{h}^{k-1}})=v_i(\hat{\sigma}|_{h^{k-1}})$, there is no uncertainty regarding agent i's choice since we assume that he prefers A to V in case of indifference. Each agent's best respond in a period is uniquely determined by the current-period inspection probability and the continuation payoffs.

A.3 Proof of Lemma 1

(i) Let

$$p = \begin{cases} f^{-1}(x) & \text{if } x \in \left[0, f\left(\frac{c}{1+c}\right)\right] \\ \frac{c}{1+c} & \text{if } x \in \left(f\left(\frac{c}{1+c}\right), \frac{1}{1-\delta}\right]. \end{cases} \tag{31}$$

Consider the following strategy that is periodic in every d periods. In each cycle, inspects on the agent (Agent i) with probability p at each of the first $d-1$ periods. If found the agent steals, inspect on him with probability 1 forever. If did not found him steal for $d-1$ periods, then in the last period of each cycle, inspect on the agent with probability 0. It's easy to verify that in each cycle, the agent A in the first $d-1$ periods and V in the dth period. Thus this periodic strategy yields the agent a discounted payoff of $\delta^{d-1} \cdot \frac{1}{1-\delta^d}$. By solving

$$\delta^{d-1} \cdot \frac{1}{1-\delta^d} = f(p) \tag{32}$$

we can find the d^* that yields the agent an expected payoff of $f(p)$.

If d^* is an integer, the problem is solved. We next deal with the case in which d^* is non-integer. Let

$$\lceil d^* \rceil = \min\{n \in \mathbb{Z} | m \geq d^*\}.$$

Consider the following inspection strategy which is in cycle for every $\lceil d^* \rceil$ periods. Inspect on the agent with probability p for $(\lceil d^* \rceil - 2)$ periods. If found violating, then inspect on him with probability 1 forever. Otherwise in the $(\lceil d^* \rceil - 1)$th period, flip an un-even coin (with probability q the coin turns out to be head and $1-q$ tail). The outcome of the coin is immediately observed by everyone. If the coin turns out to be head, then the inspector does not inspect on the agent in the current period. The new cycle starts from the next period in this case. If the coin turns out to be tail, the inspector inspects on the agent with probability p in the current period and in the next period ($\lceil d^* \rceil$th period) inspects on the agent with probability 0. We next find the value of q so that the agent's expected payoff equals to $f(p)$. The solution to

$$f(p) = q \cdot \left(\delta^{\lceil d^* \rceil - 2} + f(p) \cdot \delta^{\lceil d^* \rceil - 1}\right) + (1-q) \cdot \left(\delta^{\lceil d^* \rceil - 1} + f(p) \cdot \delta^{\lceil d^* \rceil}\right)$$

is

$$q = \frac{f(p) - \delta^{\lceil d^* \rceil - 1} - f(p) \cdot \delta^{\lceil d^* \rceil}}{\delta^{\lceil d^* \rceil - 2} + f(p)\delta^{\lceil d^* \rceil - 1} - \delta^{\lceil d^* \rceil - 1} - f(p)\delta^{\lceil d^* \rceil}},$$

and it is easy to verify that $q < 1$ (by (32)).

(ii) Following part (i), the inspector has a strategy that inspects in each period on Agent i with probability no more than $\frac{c}{1+c}$, and yields Agent i an expected payoff of $f\left(\frac{c}{1+c}\right)$. Since an agent chooses A when he is inspected with probability $\frac{1}{1+c}$, regardless of the continuation payoff, $g\left(f\left(\frac{c}{1+c}\right)\right)=0$. Since the inspector minimizes $x+g(x)$, and $g(x)\geq 0$ (see Lemma 4 (i)), the inspector is never optimal to choose an inspection strategy that yields Agent i a payoff greater than $f\left(\frac{c}{1+c}\right)$.

A.4 Proof of Proposition 4

(i) First note that if an agent chooses A in every period, he guarantees himself zero payoff. Therefore $g(x)\geq 0$ for every $x\geq 0$. To show that $g(x)$ is non-increasing, we first prove a Lemma.

Lemma 8. *Suppose the inspector has a strategy σ_0 that yields $v_1(\sigma)=z$ and $v_2(\sigma)=b$, where $z<\frac{1}{1-\delta}$. Then for every $\gamma\in(z,\frac{1}{1-\delta}]$ there exists an inspection strategy $\tilde{\sigma}_0$ of the inspector that yields $v_1(\tilde{\sigma})=\gamma$ and $v_2(\tilde{\sigma})=b$.*

Proof. Since $v_1(\sigma)<\frac{1}{1-\delta}$, there must exist a history $\hat{h}^{k-1}$ under which $\sigma_0(I_1|_{\hat{h}^{k-1}})>0$ and $\sigma_0(A|_{\hat{h}^{k-1}})=1$. That is, there exists a history $\hat{h}^{k-1}$ under which Agent 1 is inspected with positive probability and he chooses A.

Now consider another inspection strategy σ_0'. (i) $\sigma_0'(a_0|h)=\sigma_0(a_0|h)$ for $\forall h\in\cup_{t=0}^{k-1}Y^t$. (ii) $\sigma_0'(I_1|\hat{h}^{k-1})=0$, $\sigma_0'(\emptyset|\hat{h}^{k-1})=\sigma_0(\emptyset|\hat{h}^{k-1})+\sigma_0(I_1|\hat{h}^{k-1})$ and $\sigma_0'(I_2|\hat{h}^{k-1})=\sigma_0(I_2|\hat{h}^{k-1})$. (iii) $\sigma_0'|_{(\hat{h}^{k-1},\emptyset)}=\frac{\sigma_0(\emptyset|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}\cdot\sigma_0|_{(\hat{h}^{k-1},\emptyset)}+\frac{\sigma_0(I_1|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}\cdot\sigma_0|_{(\hat{h}^{k-1},A_1)}$ and $\sigma_0'|_{(\hat{h}^{k-1},y)}=\sigma_0|_{(\hat{h}^{k-1},y)}$ for $y\neq\emptyset$. (iv) $\sigma_0'|_{(h^{k-1},y)}=\sigma_0|_{(h^{k-1},y)}$ for every $h^{k-1}\neq\hat{h}^{k-1}$ and $y\in Y$.

Under σ_0', following history $\hat{h}^{k-1}$, Agent 1 is inspected with zero probability and he chooses V. In case no one is inspected in this stage, the inspector under σ_0' plays $\sigma_0|_{(\hat{h}^{k-1},\emptyset)}$ with probability $\frac{\sigma_0(\emptyset|\hat{h}^{k-1})}{\sigma_0'(\emptyset|\hat{h}^{k-1})}$, and plays $\sigma_0|_{(\hat{h}^{k-1},A_1)}$ with the remaining probability. It is easy to verify that $v_1(\sigma')>z$ and $v_2(\sigma')=b$. Denote $\nu=v_1(\sigma'|_{\hat{h}^r})$. Then any $\gamma\in[z,\nu]$ can be supported as an equilibrium under an inspection strategy $\tilde{\sigma}_0$ which randomizes between σ_0 and σ'. Lemma 8 follows by applying the same argument to each history h^r under which $\sigma_0(I_1|_{h^r})>0$ and $\sigma_0(A|_{h^r})=1$.

□

Let $z\geq 0$ and denote $\nu=g(z)$. Suppose $z'>z$. By Lemma 8, $v_1=z'$ and $v_2=g(z)$ can be supported as an equilibrium. By the definition of $g(\cdot)$, $g(z')\leq g(z)$.

(ii) Suppose, on the contrary, $g(x)$ is not convex. Then there must exist x_1, x_2 and $t\in(0,1)$ such that

$$g\big(tx_1+(1-t)x_2\big)>t\cdot g(x_1)+(1-t)\cdot g(x_2). \tag{33}$$

Let $v_1 = tx_1 + (1-t)x_2$. By the definition of $g(\cdot)$, given that Agent 1 obtains an expected payoff of v_1, there exists *no* strategy of the inspector that yields Agent 2 a payoff less than $g(v_1)$.

Consider now an inspection strategy σ_0' under which the inspector uses a random device: with probability t it turns out to be head and with the remaining probability tail. In case the outcome is head, the inspector uses an inspection strategy that yields Agent 1 x_1 and Agent 2 $g(x_1)$; if the outcome is tail he uses an inspection strategy that yields Agent 1 x_2 and Agent 2 $g(x_2)$. Under this new inspection strategy Agent 1 obtains an expected payoff of v_1, while Agent 2 obtains an expected payoff of $\big(t \cdot g(x_1) + (1-t) \cdot g(x_2)\big)$, which is less than $g(v_1)$ (by (33)), a contradiction.

(iii) Suppose $v_1 = 0$. It can be easily verified that this payoff can be supported iff in every period the inspector inspects on Agent 1 with probability no less than $\frac{1}{1+c}$. Denote $p = \frac{c}{1+c}$. The minimum payoff for agent 2 is

$$\begin{aligned} g(0) &= \delta p f(p) + \delta^2 (1-p) p f(p) + \delta^3 (1-p)^2 p f(p) + \ldots \\ &= \delta p f(p) \cdot \Big(1 + \delta(1-p) + \delta^2(1-p)^2 + \delta^3(1-p)^3 + \ldots\Big) \\ &= \frac{1-p-cp}{1-(1-p)\delta} = \frac{1-c^2}{1+c-\delta}. \end{aligned} \tag{34}$$

This is achieved by the following strategy: in the first period inspects on Agent 2 with probability $\frac{c}{1+c}$. If Agent 2 is not inspected, start from the beginning. If Agent 2 is (inspected and) found A, he obtains a continuation payoff of $f\left(\frac{c}{1+c}\right)$. If Agent 2 is found V, he is inspected with probability 1 forever. To support $f\left(\frac{c}{1+c}\right)$, the inspector uses a periodic strategy as described in the proof of Lemma 1.

We have shown that there exists an inspection strategy of the inspector under which $v_1 = 0$ and $v_2 = \frac{1-c^2}{1+c-\delta}$. Therefore $g\left(\frac{1-c^2}{1+c-\delta}\right) \leq 0$. Since $g\left(\frac{1-c^2}{1+c-\delta}\right) \geq 0$, $g\left(\frac{1-c^2}{1+c-\delta}\right) = 0$. This implies that $g(x) = 0$ for every $x \geq \frac{1-c^2}{1+c-\delta}$ (by (i)).

(iv) Suppose $x \in [0, \frac{1-c^2}{1+c-\delta})$ and $g(x) = z$. This implies that the inspector has an inspection strategy that yields Agent 1 a payoff of x and Agent 2 a payoff of z. Therefore $g(z) \leq x$. Suppose $g(z) < x$. By (i) there exists $z' < z$ such that $g(z') = x$. This implies that the inspector has a strategy that yields Agent 1 an expected payoff of x and Agent 2 an expected payoff of z', $z' < z$, a contradiction.

A.5 Proof of Proposition 5

A.5.1 Case $\frac{1-c^2}{1+c-\delta} < 1$

Consider first the case where $\frac{1-c^2}{1+c-\delta} < 1$. By Proposition 4, $g(x) < 1$ for every x. Moreover, since $g(z) = 0$ for every $z \geq \frac{1-c^2}{1+c-\delta}$, under equilibrium $f\big(\sigma_0(I_1)\big) \leq \frac{1-c^2}{1+c-\delta} < 1$. That is, under equilibrium, in a period where both agents obtain positive expected payoffs, the maximum of these two payoffs is less than $\frac{1-c^2}{1+c-\delta}(< 1)$. This excludes the possibility of giving agent 1 or 2 a reward (by not inspecting) when the other player obtains a positive payoff, since it will have a value of 1 which is larger than the amount of the total award. In other words, the inspector issues the rewards only when one of the agent is obtaining a zero payoff. In addition, $\frac{1-c^2}{1+c-\delta} < 1$ implies that both agents A in the first period (otherwise, at least one of the agent inflicts a damage of 1 on the inspector, which exceeds $0 + \frac{1-c^2}{1+c-\delta}$).

Lemma 9. *For $\forall x \in (0, \frac{1-c^2}{1+c-\delta})$, $v_1(\sigma) = x$ and $v_2(\sigma) = g(x)$ can be implemented by an inspection strategy σ_0 which satisfies $\sigma_0(\emptyset) = 0$.*

Proof. Suppose there exists a strategy $\hat{\sigma}_0$ under which $v_1(\hat{\sigma}) = x$, $v_2(\hat{\sigma}) = g(x)$ and $\hat{\sigma}_0(I_1) + \hat{\sigma}_0(I_2) < 1$. By Proposition 4 (ii) (convexity of $g(x)$),

$$\begin{aligned} & g\left(\frac{\hat{\sigma}_0(I_1)}{\hat{\sigma}_0(I_1)+\hat{\sigma}_0(\emptyset)} v_1(\hat{\sigma}|_{A_1}) + \frac{\hat{\sigma}_0(\emptyset)}{\hat{\sigma}_0(I_1)+\hat{\sigma}_0(\emptyset)} v_1(\hat{\sigma}|_{\emptyset})\right) \\ \leq & \frac{\hat{\sigma}_0(I_1)}{\hat{\sigma}_0(I_1)+\hat{\sigma}_0(\emptyset)} g\big(v_1(\hat{\sigma}|_{A_1})\big) + \frac{\hat{\sigma}_0(\emptyset)}{\hat{\sigma}_0(I_1)+\hat{\sigma}_0(\emptyset)} g\big(v_1(\hat{\sigma}|_{\emptyset})\big). \end{aligned} \tag{35}$$

Consider the following strategy σ_0^*: (i) $\sigma_0^*(I_1) = \hat{\sigma}_0(I_1) + \hat{\sigma}_0(\emptyset)$ and $\sigma_0^*(I_2) = \hat{\sigma}_0(I_2)$. (ii) $v_1(\sigma^*|_{A_1}) = \frac{\hat{\sigma}_0(I_1)}{\hat{\sigma}_0(I_1)+\hat{\sigma}_0(\emptyset)} v_1(\hat{\sigma}|_{A_1}) + \frac{\hat{\sigma}_0(\emptyset)}{\hat{\sigma}_0(I_1)+\hat{\sigma}_0(\emptyset)} v_1(\hat{\sigma}|_{\emptyset})$ and $v_2(\sigma^*|_{A_1}) = g\big(v_1(\sigma^*|_{A_1})\big)$. (iii) $\sigma_0^*|_{A_2} = \hat{\sigma}_0|_{A_2}$. Note that the existence of an inspection strategy that satisfied (ii) follows from Lemma 1. We next study agent 1's equilibrium action under σ^*. By (ii),

$$\begin{aligned} \big(\hat{\sigma}_0(I_1) + \hat{\sigma}_0(\emptyset)\big) \cdot v_1(\sigma^*|_{A_1}) &= \hat{\sigma}_0(I_1) \overbrace{v_1(\hat{\sigma}|_{A_1})}^{\geq f\left(\hat{\sigma}_0(I_1)\right)} + \hat{\sigma}_0(\emptyset) v_1(\hat{\sigma}|_{\emptyset}) \\ &\geq \hat{\sigma}_0(I_1) \cdot f\big(\hat{\sigma}_0(I_1)\big) \\ &> \sigma_0^*(I_1) \cdot f\big(\sigma_0^*(I_1)\big). \end{aligned} \tag{36}$$

The last inequality in (36) holds because $p \cdot f(p)$ is decreasing in p. Therefore $v_1(\sigma^*|_{A_1}) > f\big(\sigma_0^*(I_1)\big)$ and Agent 1 A in the first period under σ^*. It is then easy to verify that $v_1(\sigma^*) = v_1(\hat{\sigma})$ and by (35), $v_2(\sigma^*) \leq v_2(\hat{\sigma})$. In the case where $v_2(\sigma^*) < v_2(\hat{\sigma})$, we have a contradiction with $v_2(\hat{\sigma}) = g(x)$, which implies that there exists no strategy $\hat{\sigma}_0$ that yields agent 1 and 2 the payoff x and $g(x)$, respectively while satisfying $\hat{\sigma}_0(\emptyset) > 0$. If $v_2(\sigma^*) = v_2(\hat{\sigma})$, then we find an inspection strategy σ_0^* with $\sigma_0^*(\emptyset) = 0$ and $v_1(\sigma^*) = x$, $v_2(\sigma^*) = g(x)$.

$\square$

Proposition 7. *In the inspection strategy that satisfies Lemma 9,* $v_1(\sigma|_{A_1}) = f\big(\sigma_0(I_1)\big)$, $v_2(\sigma|_{A_1}) = g\big(v_1(\sigma|_{A_1})\big)$, $v_1(\sigma|_{A_2}) = g\big(v_2(\sigma|_{A_2})\big)$ *and* $v_2(\sigma|_{A_2}) = f\big(\sigma_0(I_2)\big)$.

Proof. We focus on the strategy σ_0^* that is described in the proof of Lemma 9. Denote $p_1 = \sigma_0^*(I_1)$. Since $\sigma_0^*(\emptyset) = 0$, $\sigma_0^*(I_2) = 1 - p_1$. Let $v_1 = x$, $v_2 = g(x)$, $A = v_1(\sigma^*|_{A_1})$, $B = v_2(\sigma^*|_{A_1})$, $C = v_1(\sigma^*|_{A_2})$ and $D = v_2(\sigma^*|_{A_2})$. Agents' payoffs are summarized in Figure 4.

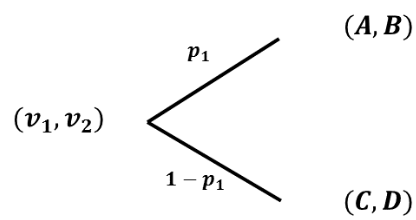

Figure 4

Since both agents A in the first period under equilibrium, $A \geq f(p_1)$ and $D \geq f(1-p_1)$.

Lemma 10. $B = g(A)$ *and* $C = g(D)$.

Proof. Since the inspector minimizes $v_1 + v_2$, $v_2 = g(v_1)$. That is, given agent 1 obtains an expected payoff $v_1 = 0 + \delta \cdot \big(p_1 \cdot A + (1-p_1) \cdot C\big)$, there exists no inspection strategy that can yields agent 2 a payoff lower than $v_2 = 0 + \delta \cdot \big(p_1 \cdot B + (1-p_1) \cdot D\big)$. By the definition of $g(\cdot)$, $B \geq g(A)$. If $B > g(A)$, then the inspector has a strategy that yields agent 1 an expected payoff of v_1, and agent 2 an expected payoff of

$$\delta \cdot \big(p_1 \cdot g(A) + (1-p_1) \cdot D\big) < v_2 = g(v_1),$$

a contradiction. Therefore $B = g(A)$. Since $v_1 = g(v_2)$, by similar argument, $C = g(D)$.

$\square$

Since $A = g\big(g(A)\big) = g(B)$, Figure 5 follows immediately from Lemma 10.

Lemma 11. *For every* $p \in (0, 1)$,

$$p \cdot g(B) + (1-p) \cdot g(D) > g\big(p \cdot B + (1-p) \cdot D\big), \tag{37}$$

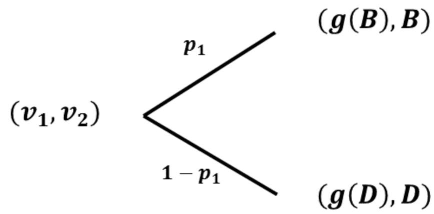

Figure 5

and

$$p \cdot g(A) + (1-p) \cdot g(C) > g\big(p \cdot A + (1-p) \cdot C\big). \tag{38}$$

Proof. By Figure 11,

$$v_1 = \delta \cdot \big(p_1 \cdot g(B) + (1-p_1) \cdot g(D)\big), \tag{39}$$

$$v_2 = \delta \cdot \big(p_1 \cdot B + (1-p_1) \cdot D\big). \tag{40}$$

Since $v_1 = g(v_2)$, by (39) - (40),

$$g\Big(\delta \cdot \big(p_1 \cdot B + (1-p_1) \cdot D\big)\Big) = \delta \cdot \big(p_1 \cdot g(B) + (1-p_1) \cdot g(D)\big). \tag{41}$$

In the case where $v_1 > 0$ (as assumed in the Proposition),

$$\delta \cdot \big(p_1 \cdot g(B) + (1-p_1) \cdot g(D)\big) < p_1 \cdot g(B) + (1-p_1) \cdot g(D). \tag{42}$$

Since $g(x)$ is non-increasing in x,

$$g\big(p_1 \cdot B + (1-p_1) \cdot D\big) \leq g\Big(\delta \cdot \big(p_1 \cdot B + (1-p_1) \cdot D\big)\Big) \tag{43}$$

By (41) - (43),

$$p_1 \cdot g(B) + (1-p_1) \cdot g(D) > g\big(p_1 \cdot B + (1-p_1) \cdot D\big). \tag{44}$$

Since $g(x)$ is (weakly) convex, by (44) inequality $p \cdot g(B) + (1-p) \cdot g(D) > g\big(p \cdot B + (1-p) \cdot D\big)$ holds for every $p \in (0,1)$ (Formal proof?).

Since $g(\cdot)$ is non-increasing, by 44,

$$g\big(p_1 \cdot g(B) + (1-p_1) \cdot g(D)\big) < g\Big(g\big(p_1 \cdot B + (1-p_1) \cdot D\big)\Big).$$

That is,

$$g\big(p_1 \cdot A + (1-p_1) \cdot C\big) < p_1 \cdot g(A) + (1-p_1) \cdot g(C).$$

□

Lemma 12. *$A = f(p_1)$.*

Proof. Suppose $(g(B) =)A > f(p_1)$. Then there must exists $p' < p_1$ such that $f(p') = g(B)$. Consider next the strategy $\tilde{\sigma}_0$: (i) $\tilde{\sigma}_0(I_1) = p'$, $\tilde{\sigma}_0(\emptyset) = p_1 - p'$ and $\tilde{\sigma}_0(I_2) = 1 - p_1$. (ii) $\tilde{\sigma}_0|_{A_1} = \tilde{\sigma}_0|_{\emptyset} = \sigma_0^*|_{A_1}$ and $\tilde{\sigma}_0|_{A_2} = \sigma_0^*|_{A_2}$. It can be verified that $v_1(\tilde{\sigma}_0) = v_1$ and $v_2(\tilde{\sigma}_0) = v_2$. This strategy is summarized in Figure 6.

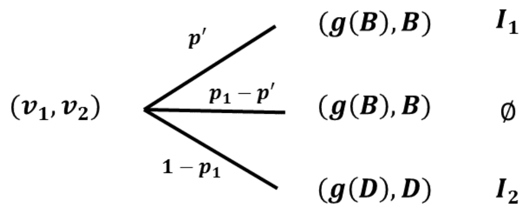

Figure 6

Finally, we show that there exists a strategy $\bar{\sigma}_0$ which yields agent 2 v_2 and agent 1 a payoff strictly lower than v_1. Consider the strategy $\bar{\sigma}_0$: (i) $\bar{\sigma}_0(I_1) = p'$ and $\bar{\sigma}_0(I_2) = 1 - p'$. (i) $v_2(\bar{\sigma}|_{A_1}) = \frac{p_1 - p'}{1 - p'} \cdot B + \frac{1 - p_1}{1 - p'} \cdot D$ and $v_1(\bar{\sigma}|_{A_1}) = g\big(v_2(\bar{\sigma}|_{A_1})\big)$. Denote $D' = v_2(\bar{\sigma}|_{A_1})$, Figure 7 summarizes this strategy.

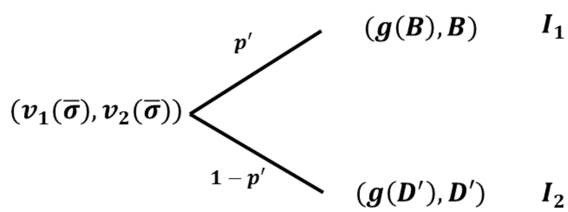

Figure 7

To compute agent 2's expected payoff under $\bar{\sigma}_0$, we first analyze agent 2's action in the first period.

$$\begin{aligned} (1 - p') \cdot D' &= (p_1 - p') \cdot B + (1 - p_1) \cdot \overbrace{D}^{\geq f(1-p_1)} \\ &\geq (1 - p_1) \cdot f(1 - p_1) \\ &> (1 - p') \cdot f(1 - p'). \end{aligned} \tag{45}$$

The second inequality holds because $p \cdot f(p)$ is decreasing in p. By (45) $D' > f(1 - p')$.

Consequently, agent 2 chooses A in the first period and his expected payoff is

$$\begin{aligned} v_2(\bar{\sigma}) &= 0 + \delta \cdot \big(p' \cdot B + (1-p') \cdot D'\big) \\ &= v_2. \end{aligned}$$

For agent 1, since $g(B) = f(p')$ (by the definition of p'), he chooses A in the first period and obtains

$$v_1(\bar{\sigma}) = 0 + \delta \cdot \big(p' \cdot g(B) + (1-p') \cdot g(D')\big). \tag{46}$$

Let $\tilde{p} = \frac{p_1 - p'}{1-p'}$. Then $D' = \tilde{p} \cdot B + (1-\tilde{p}) \cdot D$. By Lemma 11,

$$g(D') < \tilde{p} \cdot g(B) + (1-\tilde{p}) \cdot g(D). \tag{47}$$

By (46) - (47),

$$\begin{aligned} v_1(\bar{\sigma}) &< \delta \cdot \big(p' \cdot g(B) + (p_1 - p') \cdot g(B) + (1-p_1) \cdot g(D)\big) \\ &= v_1, \end{aligned} \tag{48}$$

a contradiction to $v_1 = g(v_2)$. □

Because of the symmetry of agents, by the similar argument we can prove that $D = f(1-p_1)$. This finishes the proof of Proposition 7.

□

Figure 8 follows immediately from Proposition 7.

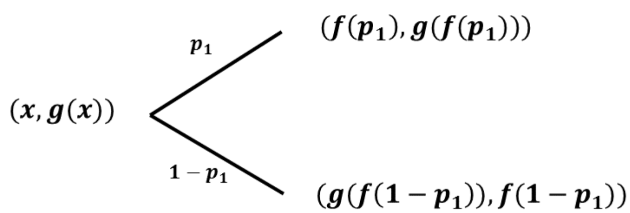

Figure 8

For every $x \in (0, \frac{1-c^2}{1+c-\delta})$,

$$x = 0 + \delta \cdot \Big(p_1 \cdot f(p_1) + (1-p_1) \cdot g\big(f(1-p_1)\big)\Big), \tag{49}$$

$$g(x) = 0 + \delta \cdot \Big(p_1 \cdot g\big(f(p_1)\big) + (1-p_1) \cdot f(1-p_1)\Big). \tag{50}$$

Denote $w(p) = p \cdot f(p) + (1-p) \cdot g\big(f(1-p)\big)$. Note that $w(p)$ is (strictly) decreasing in p for $p \in [\frac{c}{1+c}, \frac{1}{1+c}]$. Therefore the p_1 that solves (49) is $p_1^* = w^{-1}\left(\frac{x}{\delta}\right)$. This finishes the proof of Proposition 5 for $\frac{1-c^2}{1+c-\delta} < 1$.

A.5.2 Case $\frac{1-c^2}{1+c-\delta} \geq 1$

Consider next the case where $\frac{1-c^2}{1+c-\delta} \geq 1$. In this case, we need to take into account the possibility that the inspector may benefit from issuing a reward to an agent before the other agent's payoff reaches zero.

Lemma 13. *When agent 1 and 2's payoffs are $x \in [0, \frac{1-c^2}{1+c-\delta}]$ and $g(x)$, respectively, the inspector should deter at least one agent from V in the current period.*

Proof. Suppose, on the contrary, the inspector is best off inspecting on both agents with zero probability. Note first that $y \in [0, \frac{1-c^2}{1+c-\delta}]$. Then in the current period both agents obtain 1, and in the next period agent 1 and 2 obtains $\frac{x-1}{\delta}$ and $\frac{y-1}{\delta}$, respectively.

Since $x < \frac{1}{1-\delta}$ and $y < \frac{1}{1-\delta}$, $x > \frac{x-1}{\delta}$ and $y > \frac{y-1}{\delta}$. Since $g(\cdot)$ is non-increasing, $g\left(\frac{x-1}{\delta}\right) \geq g(x) = y > \frac{y-1}{\delta}$. Consequently, $g\left(\frac{x-1}{\delta}\right) > \frac{y-1}{\delta}$, a contradiction to the definition of $g(\cdot)$.

□

Proposition 8. *When agent 1 and 2's payoffs are $x \in [0, \frac{1-c^2}{1+c-\delta}]$ and $g(x)$, respectively, the inspector deters both agents from V in the current period.*

Proof. If $x < 1$ and $g(x) < 1$, Proposition 8 is straightforward since letting an agent chooses V yields him an expected payoff of 1 (which exceeds both agents payoffs). We next consider the case where at least one of the agent's payoff is no less than 1. W.l.o.g. let $g(x) \geq 1$. By Lemma 13, we only need to compare the inspector's payoff when he deters both agents from V and when he deters only one agent from V.

If the inspector deters both agents from V, the best he can do is (by similar arguments as in Proposition 7) to choose p_1 (the probability to inspect on agent 1) to be the solution of

$$x = 0 + \delta \cdot \Big(p_1 \cdot f(p_1) + (1-p_1) \cdot g\big(f(1-p_1)\big)\Big). \tag{51}$$

Agent 2's expected payoff in this case (denoted by y_1) is

$$y_1 = 0 + \delta \cdot \Big(p_1 \cdot g\big(f(p_1)\big) + (1-p_1) \cdot f(1-p_1)\Big). \tag{52}$$

Consider next the case where the inspector deters only agent 1 from Violating (therefore letting agent 2 to violate). This can be done by inspecting on agent 1 with probability 1. Then in the current period agent 1 obtains zero and agent 2 obtains 1. In the next period, agent 1 obtains an expected payoff of $\frac{x}{\delta}$. To minimize agent 2's expected payoff in the first period, his expected payoff in the next period should be $g\left(\frac{x}{\delta}\right)$. Agent 2's payoff in the first period is

$$y_2 = 1 + \delta \cdot g\left(\frac{x}{\delta}\right). \tag{53}$$

We next compare the value of y_1 and y_2. By (51), $\frac{x}{\delta} = p_1 \cdot f(p_1) + (1 - p_1) \cdot g\big(f(1 - p_1)\big)$.

$$\begin{aligned} y_2 - y_1 = 1 &+ \delta \cdot g\Big(p_1 \cdot f(p_1) + (1 - p_1) \cdot g\big(f(1 - p_1)\big)\Big) \\ &- \delta \cdot \Big(p_1 \cdot g\big(f(p_1)\big) + (1 - p_1) \cdot f(1 - p_1)\Big) \end{aligned} \tag{54}$$

Then

$$y_2 - y_1 = 1 - \delta \cdot Z, \tag{55}$$

where

$$Z = p_1 \cdot g\big(f(p_1)\big) + (1 - p_1) \cdot f(1 - p_1) - g\Big(p_1 \cdot f(p_1) + (1 - p_1) \cdot g\big(f(1 - p_1)\big)\Big). \tag{56}$$

We next show that $Z < \frac{1}{\delta}$. Since $f(p_1) \geq g\big(f(1 - p_1)\big)$,

$$f(p_1) \geq p_1 \cdot f(p_1) + (1 - p_1) \cdot g\big(f(1 - p_1)\big).$$

Therefore

$$p_1 \cdot g\big(f(p_1)\big) < g\big(f(p_1)\big) \leq g\Big(p_1 \cdot f(p_1) + (1 - p_1) \cdot g\big(f(1 - p_1)\big)\Big). \tag{57}$$

By (10),

$$(1 - p_1) \cdot f(1 - p_1) = \frac{1}{\delta} - \frac{(1 + c)(1 - p_1)}{\delta} < \frac{1}{\delta}. \tag{58}$$

By (56) - (58), $Z < \frac{1}{\delta}$. By (55), $y_2 > y_1$. Consequently, the strategy that deters *both* agents from V yields the inspector a better payoff compared with the strategy that deters only agent 1 from V. Since $y = g\big(g(y)\big) = g(x)$, by similar arguments, it is also better than the strategy that deters only agent 2 from V. Since Lemma 13 already shown that it is never optimal to let both agents to violate, Proposition 8 follows.

□

A.6 Proof of Corollary 5

By Proposition **??**, $A \geq f(p_2)$. Since $v_1 = f(p_2)$ and $v_2 = f(1 - p_2)$ can be easily implemented as an equilibrium outcome, $g\big(f(1 - p_2)\big) < f(p_2) \leq A$. By(49), $A > x$.

Similarly, $g(A) \leq g\big(f(p_2)\big) < f(1-p_2)$ and by (50) $f(1-p_2) > g(x)$. This proves (i) and (iii) of Proposition 2.

(ii) Suppose, on the contrary, $g\big(f(1-p_2)\big) \geq x$. Then $f(1-p_2) = g\Big(g\big(f(1-p_2)\big)\Big) \leq g(x)$, contradiction to (iii).

(iv) Suppose, on the contrary, $g(A) \geq g(x)$. Then $g(g(A)) \leq g(g(x))$, namely $A \leq x$, contradiction to (i).

A.7 Proof of Proposition 6

(i) Denote $x = v_1(\sigma)$. Clearly $v_2(\sigma) = g(x)$ (by the definition of $g(\cdot)$). The inspector $\min_{x\geq 0} x + g(x)$. Since $g(x)$ is convex, $g''(x) > 0$ and the inspector chooses x^* under which $g'(x)|_{x=x^*} = -1$. By the properties of $g(x)$, it can be easily verified that $g(x^*) = x^*$.

(ii) Denote $p_1 = \sigma_0(I_1)$ and $w(p) = p \cdot f(p) + (1-p) \cdot g\big(f(1-p)\big)$. Note that $w(p)$ is decreasing in p. It can be easily verified that $v_1(\sigma) = w(p_1)$ and $v_2(\sigma) = w(1-p_1)$. Since $w(p_1) = w(1-p_1)$, $p_1 = 0.5$.

A.8 Three-State Mechanism (Greenberg (1984))

In this section we compute the inspector's optimal payoff if he follows the state-dependent inspection mechanism proposed by Greenberg (1984). Note that Greenberg (1984) characterized the optimal inspection strategy when players do not discount future payoffs, while kept silence on the optimal inspection scheme even within their own model.

As proposed by Greenberg, there are three states: N (Neutral), R (Reward), and P (Punishment). The probability of being inspected in every state, as well as into which state will an individuals move if he is being inspected and (i)found A, (ii) found V.

Group	If inspected move to state		Probability of being inspected
	If S	If NS	
G_R	G_N	G_R	p_R
G_N	G_P	G_R	p_N
G_P	G_P	G_P	1

Figure 9: Optimal Inspection Strategy for Case $c = 0.7$ and $\delta = 0.9$

Under this framework, the inspector maximized his expected payoff by choosing p_R and p_N, while guarantee that players in N choose A and players in R choose V.

We denote each player's equilibrium payoff in state N, R, and P as v_i^N, v_i^R, and v_i^P, respectively, $i = 0, 1, 2$. Since agent 1 and 2 are identical, $v_1^V = v_2^V \equiv v_V$ for $V \in \{N, R, P\}$.

Consider first the agent in state R. His payoff is

$$V: \quad p_R \cdot (-c) + (1 - p_R) \cdot 1 + \delta \cdot \big(p_R \cdot v_N + (1 - p_R) \cdot v_R\big);$$

$$A: \quad 0 + \delta \cdot v_R.$$

Since there is no cheat-free mechanism and agents A in state N, $v_R > 0$ and $p_R < \frac{1}{1+c}$. That is, agents benefit from choosing V in state R and

$$v_R = p_R \cdot (-c) + (1 - p_R) \cdot 1 + \delta \cdot \big(p_R \cdot v_N + (1 - p_R) \cdot v_R\big). \tag{59}$$

Consider next the agent in state N. His payoff is

$$V: \quad p_N \cdot (-c) + (1 - p_N) \cdot 1 + \delta \cdot \big(p_N \cdot 0 + (1 - p_N) \cdot v_N\big);$$

$$A: \quad 0 + \delta \cdot \big(p_N \cdot v_R + (1 - p_N) \cdot v_N\big).$$

The agent is best off choosing A iff

$$p_N \geq \frac{1}{1 + c + \delta \cdot v_R}, \tag{60}$$

in which case he obtains

$$v_N = 0 + \delta \cdot \big(p_N \cdot v_R + (1 - p_N) \cdot v_N\big). \tag{61}$$

By (59) and (61),

$$v_R = \frac{(\delta \cdot p_N + 1 - \delta) \cdot (1 - c \cdot p_R - p_R)}{\delta \cdot p_N \cdot (1 - \delta) + \delta \cdot p_R \cdot (1 - \delta) + (1 - \delta)^2}, \tag{62}$$

$$v_N = \frac{\delta \cdot p_N \cdot (1 - c \cdot p_R - p_R)}{\delta \cdot p_N \cdot (1 - \delta) + \delta \cdot p_R \cdot (1 - \delta) + (1 - \delta)^2} \tag{63}$$

Since the two agents are independent and identical, we next compute the inspector's expected loss, from agent 1. Recall that agent 1 A in state N and he V in state R.

$$v_0^R = -1 + \delta\big(p_R \cdot v_0^N + (1 - p_R) \cdot v_0^R\big), \tag{64}$$

$$v_0^N = 0 + \delta \cdot \big(p_N \cdot v_0^R + (1 - p_N) \cdot v_0^N\big). \tag{65}$$

By (64) and (65),

$$v_0^R = -\frac{\delta \cdot p_N + 1 - \delta}{\delta \cdot p_N \cdot (1-\delta) + \delta \cdot p_R \cdot (1-\delta) + (1-\delta)^2}, \tag{66}$$

$$v_0^N = -\frac{\delta \cdot p_N}{\delta \cdot p_N \cdot (1-\delta) + \delta \cdot p_R \cdot (1-\delta) + (1-\delta)^2}. \tag{67}$$

Since $v_0^N > v_0^R$, the inspector is best off with putting agent 1 in state N. Therefore the inspector chooses p_N and p_R to maximize v_0^N. By (66), the inspector should minimize p_N and maximizes p_R. By 60,

$$p_N = \frac{1}{1 + c + \delta \cdot v_R}. \tag{68}$$

Since v_R is decreasing in p_R (by 62), p_N is increasing in p_R.

For every $p_R \in (0, \frac{1}{1+c})$, by solving (62) and (68) we can compute the optimal $p_N(p_R)$. Replacing the p_N in (67) with $p_N(p_R)$ we can find the optimal p_R that minimizes v_0^N.

It can be shown that in case $c = 0.7$ and $\delta = 0.9$, $p_R = 0.294$, $p_N = 0.24$, and $v_0^N = -3.728$. The inspector's minimum loss from both agents in the Greenberg-type mechanism is therefore $2v_0^N = -7.456$.